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VIABILITY OF BOUNDARY OF THE VIABILITY KERNEL

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Abstract. Viability theory allows the study of the dynamics of a system described through a set-valued map F and for which we look for trajectories which remain in a subset K. These are called *viable solutions* of the system. When F is upper semicontinuous, viability theorems state that, under some assumptions, for any initial state x_0, there exists a solution starting at x_0 and this solution is viable in K if and only if K is a viability domain; i.e., $\forall\, x \in K$, $F(x) \cap T_K(x) \neq \emptyset$. When K is not a viability domain, we study the largest viability domain of K, $Viab_K(F)$. We prove that its boundary enjoys the property of local viability at any point of the interior of K.

1. Introduction and definitions. If we know numerous results about local or global existence of solutions to differential inclusions and their viability in a subset (see Haddad [9], Aubin [2], [4], [5], Frankowska [4], [8]), and their approximation (see Aubin [6], Saint-Pierre [13], [14]), we are faced with new questions when we study the set of solutions, the target problem or the invariant and viability domains of a subset K associated to a system described by the differential inclusion:

$$\begin{cases} \dot{x}(t) \in F(x(t)), \quad \text{for almost all } t \geq 0, \\ x(0) = x_0 \in K, \\ x(t) \in K, \quad \forall\, t \geq 0, \end{cases} \tag{1}$$

where F is a set-valued map defined from a closed subset K of a finite dimensional vector space X to X and $K \subset \mathrm{Dom}\,(F)$, a closed subset of X.

We denote by $S_F(X)$ the set of solutions to the differential inclusion:

$$\dot{x}(t) \in F(x(t)), \quad \text{for almost all } t \geq 0, \quad x(0) = x; \tag{2}$$

$$S_F(X) = \left\{ x(\cdot) \in W^{1,1}(0, +\infty; X, e^{-bt}dt) : \ x(\cdot) \text{ is a solution of } (2) \right\},$$

where $W^{1,1}(0, +\infty; X, e^{-bt}dt)$ is the set of absolutely continuous functions defined by

$$\left\{ x(\cdot) \in L^1(0, +\infty; X, e^{-bt}dt) : \ \dot{x}(\cdot) \in L^1(0, +\infty; X, e^{-bt}dt) \right\}$$

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