Differential and Integral Equations, Volume 4, Number 6, November 1991, pp. 1133–1146.

EXISTENCE OF SOLUTIONS OF ELLIPTIC EQUATIONS INVOLVING CRITICAL SOBOLEV EXPONENTS WITH NEUMANN BOUNDARY CONDITION IN GENERAL DOMAINS*

M. Comte

Laboratoire d'Analyse Numérique, Tour 55-65, 5^e^ étage, Université Pierre et Marie Curie
4, Place Jussieu, 75252 Paris Cedex 05, France

M.C. Knaap†

Mathematical Institute, Leiden University, P.O. Box 9512, 2300 RA Leiden, The Netherlands

(Submitted by H. Brezis)

1. Introduction. We consider the following problem:

$$\text{(I)} \qquad \begin{cases} -\Delta u = |u|^{p-1}u + \lambda u & \text{in } \Omega, \qquad (1.1)\\ \dfrac{\partial u}{\partial n} = 0 & \text{on } \partial\Omega, \qquad (1.2)\end{cases}$$

where Ω is a bounded domain of class C^2 in $\mathbb{R}^N$, $N \geq 3$, n is the outward pointing normal on $\partial\Omega$, $\lambda \in \mathbb{R}$ and $p = (N+2)/(N-2)$ is the critical Sobolev exponent for the embedding

$$H^1(\Omega) \to L^{p+1}(\Omega)$$

which is no longer compact.

Remark that $u \equiv 0$ is always a solution of problem (I) and that when $\lambda < 0$, $u = \pm|\lambda|^{1/(p-1)}$ is a solution of problem (I); these solutions will be called trivial solutions of problem (I). Our purpose is to find nontrivial solutions of problem (I).

When Ω is a ball, radially symmetric solutions of this problem have been studied in [3], [11], [17] and [22], and when $3 \leq N \leq 6$, the existence of radial solutions is proved if λ belongs to certain intervals.

However, we have proved in [15], that when Ω is a ball, then for every $\lambda \in \mathbb{R}$ there exist infinitely many nontrivial solutions of problem (I), which are not radial.

Adimurthi and Mancini, on one hand, and X.-J. Wang on the other hand, have shown that there exists a constant $\lambda(\Omega) < 0$ such that for every $\lambda < \lambda(\Omega)$, problem (I) admits a positive nontrivial solution; see [1] and [21].

In this paper we are interested in the existence of solutions of problem (I) for every $\lambda \in \mathbb{R}$. If we integrate (1.1) over Ω and insert the boundary condition (1.2), we obtain that for $\lambda \geq 0$, there are no positive solutions of problem (I). Since we

---

Received for publication December 15, 1990.

*This work was supported by the Paris VI-Leiden exchange program.

†Supported by the Netherlands organization for scientific research NWO, undcr number 611-306-016.

AMS Subject Classifications: 35J60.