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# POINCARÉ-BENDIXSON THEORY FOR CERTAIN RETARDED FUNCTIONAL DIFFERENTIAL EQUATIONS

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**Abstract.** Sufficient conditions are obtained for the absence of chaotic motion and for the existence of an orbitally stable periodic trajectory of autonomous retarded functional differential equations expressed in the feedback control form. The main condition is an inequality of frequency domain type which restricts the size of the Lipschitz constant of the non-linear part of the equation. A detailed study is made of delayed symmetric Goodwin equations of dimensions 3, 4.

**1. Introduction.** This paper concerns nonlinear autonomous retarded functional differential equations in $\mathbb{R}^n$. Its main aim is to provide a new method for proving the absence of chaotic motions and the existence of at least one orbitally stable periodic trajectory.

For proving the existence of periodic trajectories of autonomous retarded functional differential equations two main methods are available, namely fixed point theory and bifurcation theory. The torus principle for ordinary differential equations was transformed by Jones [8], Nussbaum [11], Chow and Hale [2] and many others into a method which obtains a closed trajectory by applying a fixed point theorem to a mapping of a convex set into itself. This is a powerful method but it provides no information about the stability of the closed trajectory which it produces. The Hopf bifurcation theorem, as generalised by Chow and Mallet-Paret [3] and others, can prove the existence of a small closed trajectory near a critical point whose stability has been changed by a small change of parameter. This method can also determine the stability of the closed trajectory which it produces though heavy computation is usually necessary (see [17]). For less restricted parameter values, global bifurcation theorems given by Nussbaum [12] and others can prove the existence of a closed trajectory but not its stability. A useful discussion of the above methods and their history is given by Hale [6, Chapter 11].

For certain special equations with distributed delays which arise in mathematical biology, MacDonald [9] and others have used a formal transformation to reduce the equation to an ordinary differential equation of higher dimension. The torus principle can then be used to prove the existence of a closed trajectory but not its stability. Another new method for proving the existence of a closed trajectory was given by Mallet-Paret [10] who used an ingenious Lyapunov function to obtain

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