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VOLTERRA INTEGRO-DIFFERENTIAL INEQUALITY AND ASYMPTOTIC CRITERIA

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Abstract. We give some new general results on asymptotic stability of zero solution of the nonlinear Volterra integro-differential equation (1.1), under the assumption that there exists a Liapunov function satisfying $D^+v(t) \leqq -\alpha v(t) + \int_0^t \omega(t,s)v(s)\,ds$.

1. Introduction. The purpose of this paper is to study the asymptotic behavior of solutions of the nonlinear Volterra integro-differential equation

$$x'(t) = F(t, x(t)) + \int_0^t G(t, s, x(s))\,ds, \tag{1.1}$$

where $F(t,x)$ and $G(t,s,x)$ are continuous n-vectors for $t \geqq s \geqq 0$ and $x \in S_H$, and $F(t,0) = G(t,s,0) = 0$ for all $t \geqq s \geqq 0$.

Some investigators have studied equations of this type, especially the type of the system

$$x'(t) = Ax(t) + \int_0^t C(t,s)x(s)\,ds, \tag{1.2}$$

where A is an $n \times n$ constant matrix and $C(t,s)$ is a continuous $n \times n$ matrix defined for $t \geqq s \geqq 0$.

To study the asymptotic behavior of solutions of (1.2), several techniques have been developed. For example, Grimmer and Seifert [4] studied it by the method of Liapunov-Razumikhin when A is a stable matrix. Miller [6] investigated it by using the resolvent when $C(t,s)$ is of convolution type. Burton [1], Burton and Mahfoud [3] have also studied stability properties of (1.2) by means of constructing various Liapunov functionals. These results are summarized in [2]. Recently, Hara, Yoneyama and Itoh [5] consider the system

$$x'(t) = A(t)x(t) + \int_0^t G(t, s, x(s))\,ds, \tag{1.3}$$

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