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ON A STEADY STATE FREE BOUNDARY PROBLEM ARISING IN COMBUSTION THEORY

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Abstract. The existence of a weak solution of a two-dimensional elliptic free boundary problem arising in the theory of flame propagation is shown. The model of the flame front considered is the one derived by Buckmaster.

The purpose of this paper is to establish the existence of a weak solution of an elliptic free boundary problem arising in the theory of flame propagation. The flame front model considered in this paper is the one derived by Buckmaster in [2] using asymptotic analysis in the limit of large activation energy. The density is assumed to be constant.

There are few mathematical works on the existence of solutions of free boundary problems arising in the theory of combustion. In [10] we treated the time dependent case and in [4] Crowley studied the numerical solution of a time dependent one dimensional free boundary problem of a slender tip in a pre-mixed flame. So far the elliptic case has been an open problem (cf. [2], p. 490) and the result obtained in this paper seems new.

In Section 1 we state the main assumptions and reformulate the problem in terms of weak solutions. A nonlinear elliptic boundary problem is considered in Section 2. A mixed boundary problem for a second order linear elliptic equation in a piecewise smooth domain is considered in Section 3. The proof of the main result is carried out in Section 4.

Section 1. Let Γ_+ be a smooth simple curve in the upper half of the plane, intersecting the x_1- axis at $P_\pm$ with contact angles $\alpha_\pm \neq \pi/2$. The open subset of $\mathbb{R}^2$ bounded by Γ_+ and by the interval $I = [P_-, P_+]$ is denoted by G.

$W^{k,p}(G)$ is the Banach space

$$W^{k,p}(G) = \{u : D^\alpha u \in L^p(G),\ |\alpha| \leq k\}$$

with the norm

$$\|u\|_{W^{k,p}(G)} = \left\{\sum_{|\alpha|\leq k} \|D^\alpha u\|^p_{L^p(G)}\right\}^{1/p}, \quad 1 < p < \infty.$$

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