WEAK FLOW-INVARIANCE FOR NON-CONVEX DIFFERENTIAL INCLUSIONS

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Abstract. A local existence result is proved for the viability problem

$$\dot{x}(t) \in F(t, x(t)), \quad x(0) = x_0, \quad x(t) \in K \quad \text{for all } t \in [0, \delta],$$

where F is a Carathéodory lower semicontinuous, integrably bounded, closed valued multi-function and K is a locally compact subset of a reflexive Banach space.

1. Introduction. The existence theory for the initial value problem

$$\dot{x}(t) \in F(t, x(t)), \quad x(0) = x_0, \tag{1}$$

with F a Carathéodory lower semicontinuous multifunction defined on an open (t, x)-set, is well established (see [1], [12]), even in Banach spaces and without the convexity assumption on its values. The weak flow-invariance problem for (1), or, according to Aubin's definition, the viability problem (that is the existence of at least a solution of (1) which remains all the time in a prescribed closed set K), has also been solved (see [2], [3]) when F is jointly lower semicontinuous. Recently, V.V. Goncharov [9] developed an original fixed point argument, in order to handle the Carathéodory case. However, his result requires the convexity of the set K, and, thanks to a counterexample due to A. Bressan (see [3], §7), it seems very hard to extend his technique to a non-convex K.

In [3] and [7] a new method to treat (1) when F is jointly lower semicontinuous and bounded was developed. The authors first consider a class of discontinuous single-valued maps (called directionally continuous) for which the Cauchy problem

$$\dot{x}(t) = f(t, x(t)), \quad x(0) = x_0, \tag{2}$$

admits solutions, and then prove that every lower semicontinuous, closed valued, bounded multifunction admits a directionally continuous selection. If f is such a selection from F, then every solution of (2) is also a solution of (1). This way, many qualitative results concerning lower semicontinuous differential inclusions were obtained (see [4], [5]).

In this paper, the technique of directionally continuous selections is adapted to the Carathéodory, integrably bounded case by using the Scorza Dragoni property and