Differential and Integral Equations, Volume 5, Number 1, January 1992, pp. 165–172.

PERIODIC BOUNCING FOR A FORCED LINEAR SPRING WITH OBSTACLE

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1. Introduction. The purpose of this paper is to consider periodic motions for two related problems. The first is a situation where a particle is held in place by a spring with spring constant b. The spring is under tension, which presses the particle from the positive direction against a barrier at $x = 0$. The particle is subject to a small periodic forcing term.

In the normal state of affairs, the particle will remain at rest at $x = 0$. If the particle is given a small amplitude forcing term in the positive direction, then the spring pushes it back against the barrier and the particle bounces elastically against the barrier. In this situation, we are interested in whether it is possible for large amplitude T-periodic bouncing to occur in the presence of a small amplitude forcing term and small viscous damping. In other words, can a single large amplitude blow cause the particle to break loose from its secured position and cause it to engage in (presumably destructive) voilent bouncing.

It is easy to envisage this type of bouncing occurring in satellite antennae which are hinged, and which are held in a fully extended position by a spring under pressure.

A related physical problem occurs if we consider a particle of mass one attached to a spring. As long as we remain near the equilibrium position, say at $x = 1$, the particle obeys the usual linear spring equation. Now assume that at $x = 0$, the particle may bounce elastically against a barrier (for example if the spring is fully compressed). Now, if the particle is subject to a small forcing term, and subject to small viscous damping, we expect the system to respond with a small linear T-periodic motion. One can ask, however, if in addition, there exist *large amplitude* T-periodic solutions for the same data.

Thus, in either case, we are led to study T-periodic solutions of the equation

$$\begin{aligned} &\text{(i)} \quad u'' + \delta u' + bu = \pm 1 + \varepsilon \sin t \\ &\text{(ii)} \quad u \geq 0 \\ &\text{(iii)} \quad u(t_0) = 0 \Rightarrow u'(t_0+) = -u'(t_0-). \end{aligned} \tag{1}$$

Received August 1990.
†Research supported in part by grant DMS-8722593 from the National Science Foundation.
‡Research supported in part by grant DMS-8722532 from the National Science Foundation.
AMS Subject Classifications: 34B.