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REGULARIZATION OF PARTIAL COLLISIONS IN THE N-BODY PROBLEM

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Abstract. We study solutions of the N-body problem leading to a simultaneous collision of n bodies, $2 \leq n \leq N$. By suitable coordinate and time transformations, the equations of motion are regularized relative to this kind of collision. The solution is globally defined in the new time variable. Some qualitative results concerning the ultimate behavior of the partial collapse solutions are found with the aid of the transformed equations of motion. A certain set of equilibrium points is further proved to be attractive for the partial collision orbits and we show that these solutions belong to the unstable sets (manifolds, if $n = N$) associated to the above equilibria, obtaining also the existence of the studied orbits. Then, using different transformations for the standard equations of motion, we prove that sufficiently hyperbolic partial collapse solutions can be C^1-regularized with respect to time; i.e., the motion has an extension, as a function of class C^1, beyond the considered collision singularity.

1. Introduction. Generally speaking the term *regular* describes a *normal* situation, process or phenomenon having a *good* behavior in a specified sense. It is therefore natural to think that *to regularize* means to eliminate or to change an unusual, singular appearance in order to make it *natural* and *convenient* for certain purposes.

The notion of *regularization* in the N-body problem is used in order to describe two different processes having a certain connection between them: *the regularization of the solution* and *the regularization of the equations of motion.* The first one is related with the problem of finding, for given initial conditions, a solution of the N-body problem, in the form of a power series convergent for all time. This was done in a certain sense in the three-body problem by Sundman [39]. In order to elude the case of total collapse (by imposing the condition $\mathbf{c} \neq \mathbf{0}$, where $\mathbf{c}$ is the *angular momentum constant*), the main difficulty he had to overcome was that of the double collisions. More precisely, the solution is defined in this case on some maximal interval $[0, t^*)$, with t^* finite (called *singularity of the solution*), the collision arising at moment t^*. In order to fulfill his program for all initial conditions and to express the solutions leading to double collisions as power series convergent for all time, Sundman succeeded in extending the orbit beyond the collision, as an analytic function, which physically corresponds to an elastic bounce. This process is called *analytic regularization of the solution with respect to time.* Sundman's

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