Differential and Integral Equations, Volume 5, Number 1, January 1992, pp. 95–101.

POSITIVE SOLUTIONS OF ELLIPTIC NON-POSITONE PROBLEMS*

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(Submitted by: A.R. Aftabizadeh)

Abstract. We give conditions for the existence or nonexistence of positive solutions of second-order subcritical elliptic nonpositone problems. We do not assume that the problems are radial, nor that they satisfy a variational structure. Our chief tools are Degree Theory, *a priori* estimates, and Maximum Principle arguments.

In this paper we are interested in the existence or non-existence of positive classical solutions for the problem:

$$\left.\begin{aligned} \ell u &= -\Delta u + 2\Sigma b_j D_j u = \lambda f(x,u) && \text{in} \quad \Omega \\ u &= 0 && \text{on} \quad \partial\Omega \end{aligned}\right\} \tag{1}$$

in a smooth bounded domain $\Omega \subset \mathbb{R}^n$. Here we assume that $b_j \in C^\alpha(\overline{\Omega})$, $f \in C^\alpha_{\ell oc}(\mathbb{R}^{n+1})$ with f superlinear and subcritical: $f(x,\xi) \sim \xi^\gamma$ for $0 < \xi$ large, with $1 < \gamma < (n+2)/(n-2)$. Of specific interest to us is the nonpositone situation, $f(x,0) < 0$ and the prototype equation is:

$$\left.\begin{aligned} -\Delta u &= \lambda[u^\gamma - \varepsilon] && \text{in} \quad \Omega \\ u &= 0 && \text{on} \quad \partial\Omega \end{aligned}\right\}. \tag{2}$$

Unlike in the usual case, $f(x,0) \geq 0$, there seems to be relatively little literature for this situation. We mention in particular the existence criteria of Castro and Shivaji [5, 6], and Smoller and Wasserman [14]. Furthermore, a variational existence result similar to what we shall establish here may be found in Chapter 3 of the thesis of S. Unsurangsie [18]. We thank the referee for bringing this reference to our attention. Nonexistence conditions for λ large may be found in [4] for the radial case. We recall that nonpositone radial problems are of interest if one considers

Received October 90, in revised form April 91.
*Research supported by NSERC (Canada) and CNR (Italy).
AMS Subject Classifications: 35J60, 35J65.