Differential and Integral Equations, Volume 5, Number 1, January 1992, pp. 73–93.

STEADY AND EVOLUTION STOKES EQUATIONS IN A POROUS MEDIA WITH NON-HOMOGENEOUS BOUNDARY DATA: A HOMOGENIZATION PROCESS

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(Submitted by: J.L. Bona)

Abstract. In this paper, we study the homogenization of the steady state and evolution Stokes equations with nonhomogeneous Dirichlet data on the boundary of the holes of a porous media Ω_ε, obtained from a domain Ω by removing a large number of holes of size ε ($\varepsilon > 0$, a small parameter), periodically distributed with period ε. In the homogenization process, we obtain a well defined system of equations involving both the 'slow' variable x and the 'fast' variable $y = \frac{x}{\varepsilon}$. We also derive the Darcy's law which contains an extra term and this additional term is the contribution due to the non-homogeneous data.

1. Introduction and the problem to be studied. We consider the steady state and evolution Stokes equation in a porous domain Ω_ε which is obtained from a domain Ω by removing a large number of holes of size ε (a small positive parameter) periodically distributed in the domain with period ε. We study the homogenization of the Stokes system with non-homogeneous Dirichlet condition on the boundary of the holes.

First we introduce the standard notations and then formulate the problems to be treated in this paper.

Notations. Let $Y = (0,1)^N$, $N \geq 2$, and T be an open set strictly contained in Y with smooth boundary S (the boundary S is a smooth manifold of dimension $N-1$) and $Y^* = Y \setminus \overline{T}$. Let $k \in \mathbb{Z}^N$, where $\mathbb{Z}$ is the set of all integers, and let

$$Y_k = Y + k, \quad T_k = T + k, \quad Y_k^* = Y^* + k, \quad S_k = S + k = \partial T_k.$$

Let $\Omega \subset \mathbb{R}^N$ be a bounded domain with smooth boundary Γ. Let $\varepsilon > 0$ be a small positive parameter. Consider the index sets

$$I_\varepsilon = \left\{k \in \mathbb{Z}^N : \varepsilon Y_k \subset \Omega\right\} \quad \text{and } J_\varepsilon = \left\{k \in \mathbb{Z}^N : \varepsilon Y_k \cap \Gamma \neq \emptyset\right\}.$$

Loosely speaking, $\{\varepsilon T_k, k \in I_\varepsilon\}$ are interior holes and $\{\varepsilon T_k : k \in J_\varepsilon\}$ are boundary holes and then define the perforations in Ω as follows:

$$T_\varepsilon = \bigcup_{k \in I_\varepsilon} \varepsilon T_k, \quad S_\varepsilon = \partial T_\varepsilon = \bigcup_{k \in I_\varepsilon} \partial\left(\varepsilon T_k\right).$$

Received October 1990.
AMS Subject Classifications: 35B27, 35B40, 73B27, 76D30.