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PERRON'S METHOD FOR MONOTONE SYSTEMS OF SECOND-ORDER ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

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1. Introduction. Recently the study of systems of fully nonlinear, degenerate elliptic PDEs has been undertaken by several authors [1, 6, 9, 10, 14, 16, 20] in the framework of viscosity solution. It is now known [10] that Perron's method extends to quasi-monotone systems and it, together with fixed point theorems, produces continuous viscosity solutions for monotone systems. The adaptation of Perron's method to quasi-monotone systems is rather straightforward and, indeed, not more than a componentwise application of Perron's method to scalar equations as explained implicitly in [9].

The objective here is to introduce a new adaptation of Perron's method to systems which are not necessarily quasi-monotone. The new method directly yields continuous viscosity solutions for monotone systems, without appealing to any fixed point theorem, when uniqueness of viscosity solutions is available.

This paper is organized as follows. In Section 2, we give the definition of viscosity solutions for systems of fully nonlinear, second-order, elliptic PDEs in diagonal form. In Section 3, we generalize Perron's method so as to apply to such systems. In Section 4, we establish uniqueness results for multi-valued viscosity solutions and an existence result for continuous viscosity solutions under a monotonicity and a regularity assumption on the systems. In Section 5, we reformulate the monotonicity and the regularity assumption and generalize the uniqueness and existence results of Section 4. Section 6 concerns two examples of systems which illustrate the applicability of our uniqueness and existence results.

2. Viscosity solutions for systems. We are concerned with the system of nonlinear second-order elliptic PDEs

$$\begin{cases} F_1(x, u(x), Du_1(x), D^2u_1(x)) = 0 & \text{in } \Omega, \\ F_2(x, u(x), Du_2(x), D^2u_2(x)) = 0 & \text{in } \Omega, \\ \vdots \\ F_m(x, u(x), Du_m(x), D^2u_m(x)) = 0 & \text{in } \Omega. \end{cases} \tag{2.1}$$

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