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P-SYMMETRIES OF TWO-DIMENSIONAL P-F VECTOR FIELDS

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Abstract. We calculate the group of p-symmetries for each Bass-Meisters normal form of two-dimensional p-f vector fields. We also examine the behavior of the group of p-symmetries under coordinate change. We deduce that we have found the group of p-symmetries of each two-dimensional p-f vector field up to conjugation by a polyomorphism.

1. Introduction.

Consider the initial value problem

$$\dot{y}\ (\equiv \frac{dy}{dt}) = \mathbf{V}(y), \qquad y(0) = x \in \mathbb{F}^n, \tag{1.1}$$

where $\mathbf{V}$ is a continuously differentiable vector field on $\mathbb{F}^n$ ($\mathbb{F}$ is $\mathbb{R}$ or $\mathbb{C}$). Let $\phi\colon \Omega \to \mathbb{F}^n$ be the (local) flow associated with (1.1) where Ω, an open subset of $\mathbb{R} \times \mathbb{F}^n$, is the maximal domain of ϕ. For each t in $\mathbb{R}$ let U^t be the set of all x in $\mathbb{F}^n$ such that (t, x) is in Ω. The flow ϕ is said to be a *polynomial flow* and $\mathbf{V}$ is said to be a *p-f vector field* if for each t in $\mathbb{R}$ the t-advance map $\phi^t\colon U^t \to \mathbb{F}^n$ is polynomial. That is, if for each t in $\mathbb{R}$, each component of ϕ^t is polynomial.

Call a polynomial map $P\colon \mathbb{F}^n \to \mathbb{F}^n$ a *polyomorphism* (short for *poly*nomial aut*o-morphism*) if P has a polynomial inverse. Call a diffeomorphism $F\colon \mathbb{F}^n \to \mathbb{F}^n$ a *symmetry* of the vector field $\mathbf{V}$ if $F'(x)\mathbf{V}(x) = \mathbf{V}(F(x))$ for all x in $\mathbb{F}^n$. (We identify $F'(x)$ with the matrix whose ijth entry is $\partial F_i/\partial x_j$.) Equivalently, a diffeomorphism $F\colon \mathbb{F}^n \to \mathbb{F}^n$ is a symmetry of $\mathbf{V}$ if F sends solutions of (1.1) to solutions. If $P\colon \mathbb{F}^n \to \mathbb{F}^n$ is both a polyomorphism and a symmetry, call P a *p-symmetry* of $\mathbf{V}$. The main result in this paper is a "complete" description of the set of p-symmetries of each two-dimensional p-f vector field. Our description is based on Theorem 11.8 of Bass and Meisters [3] which gives a set of normal forms, the *Bass-Meisters normal forms*, for two-dimensional p-f vector fields under polyomorphic coordinate changes. (See also Zurkowski [16].) Their Theorem 11.8 is restated here, along with a list of their normal forms, as our Theorem 4.1. In this paper we describe how the set of p-symmetries behaves under a coordinate change and then we calculate the set of

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