Differential and Integral Equations, Volume 5, Number 2, March 1992, pp. 421–432.

MAXIMUM NORM IN ONE-DIMENSIONAL HYPERBOLIC PROBLEMS

RONALD GRIMMER†
Department of Mathematics, Southern Illinois University, Carbondale, IL 62901

EUGENIO SINESTRARI‡
Dipartimento di Matematica, Università di Roma "La Sapienza", 00185 Roma, Italy

(Submitted by: Franz Kappel)

Abstract. Well posedness in a space of continuous functions is proved for one-dimensional symmetric hyperbolic systems and maximum norm estimates are provided for solutions and their derivatives. An extensive application is made to the Cauchy-Dirichlet problem for the wave equation in a bounded interval.

I. Introduction. In this paper, we study a class of hyperbolic problems and, in particular, the Cauchy-Dirichlet problem for the nonhomogeneous wave equation; the main feature of our research consists in the fact that we find classical solutions; i.e., verifying the equations pointwise; this will be a consequence of our use of the Banach space of continuous functions and its maximum norm (instead of the usual L^2 norm). Since a result of W. Littman in [5] proves the nonexistence of L^p estimates for the n-dimensional wave equation when $n > 1$ and $p \neq 2$ (see also its generalization to symmetric hyperbolic systems given by P. Brenner in [1]), we must consider only the one-dimensional problems.

In our approach, we reduce the partial differential problem to an ordinary differential one in a certain Banach space X of continuous functions of the space variable:

$$\begin{cases} U'(t) = AU(t) + F(t), & t \in [0,T] \\ U(0) = U_0, \end{cases} \tag{1.1}$$

where $A : D(A) \subseteq X \to X$ is a closed linear operator. In some important applications (e.g., in the Cauchy problem for the wave equation with homogeneous Dirichlet conditions in a finite interval), $D(A)$ is not dense in X and so, according to the Hille-Yosida theorem (see e.g. [6]), A does not generate a semigroup and (1.1) cannot be solved by the variation of constants formula; for this reason we will use a theorem which gives conditions for the existence of a unique classical solution of (1.1) also when $\overline{D(A)} \neq X$.

Received September 1990.
†Work partially supported by NSF grant DMS-8906840.
‡Work supported by the Italian M.P.I. project "Equazioni di evoluzione e applicazioni".
AMS Subject Classifications: 47D05, 35L05, 35L50.