Differential and Integral Equations, Volume 5, Number 2, March 1992, pp. 405–420.

A FUNCTIONAL EQUATION FROM CELL KINETICS

ANNETTE GRABOSCH†
LB Biomathematik, Biologie II, Universität Tübingen
Auf der Morgenstelle 10, D–7400 Tübingen, Germany

(Submitted by: A.R. Aftabizadeh)

Abstract. We present a model from biology, more precisely from cell biology, in which the evolution of a cell population throughout the cell cycle is described by means of an abstract functional equation. The solutions can be considered as the orbits of a one–parameter semigroup of positive operators. The asymptotic behavior of the cell population will be discussed using functional analytic tools, such as the theory of one–parameter semigroups of operators, the theory of positive operators on Banach lattices, especially on spaces of integrable functions, and the Perron–Frobenius spectral theory. Compactness properties and strong positivity properties of the solution semigroup, such as irreducibility, play an essential role in the analysis of the cell population equation.

1. Introduction. We consider a cell cycle model proposed by M. Kimmel, Z. Darzynkiewicz, O. Arino and F. Traganos [10] (see also [1] and [15]). There is already an extensive literature treating this model. The first biological formulation of the model is presented in [10]. A thorough mathematical analysis is given in [1]. In these last two references, one can also find a biological derivation and discussion of the model. In succession to these first investigations, there has been a series of generalizations. Recently, O. Arino and M. Kimmel (see e.g., [2]) discussed some nonlinear variants of the model. Furthermore, we refer to [19], [11], [8], [13] and [18] for related investigations.

We start this paper with a short review of the basic linear cell division model. Then we will give a outline of the mathematical treatment using recent functional analytic tools. Additionally, we add one new result to the investigation given in [1]. O. Arino and M. Kimmel assume that the size of individual cells of the population (after some time $t > 0$) is always greater than a fixed value $A > 0$ and less than a fixed value $B < \infty$. In other words, the size of cells is restricted to a fixed compact interval in $(0, \infty)$. Using this assumption, they study the asymptotic behavior of the population. They actually prove that the size distribution of the population converges to an exponential steady state distribution. In our approach, especially in our main result (cf. Theorem 2.2), we do not restrict the size of individual cells to some size interval (compare with [1], §2, Theorem). We actually do allow cells of arbitrarily small or arbitrarily large size in our initial population. Nevertheless, during the study the dynamical behavior of the population size and the size distribution of the limit population, it turns out that the size of the individual cells in the asymptotic distribution will automatically be bounded from above and below.

Received October 1990.
†The author was supported in part by a grant from "Deutsche Forschungsgemeinschaft" DFG.
AMS Subject Classifications: 92A15, 47D05.