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SOME STABILITY RESULTS FOR PERTURBED SEMILINEAR PARABOLIC EQUATIONS

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Abstract. We prove a nonlinear variation of constants formula for a class of semilinear parabolic equations, and use this formula to study the stability of nonlinear perturbations of such equations. We give examples in which the theorems proved determine the stability of a perturbation of a nonuniformly stable equation, and where the application of converse Lyapunov theorems is not possible.

1. Introduction and preliminaries. Our goal in this paper is to prove a representation formula and some stability results for a parabolic evolutionary equation of the form

$$u' + Au = F(t,u) + G(t,u) \tag{1.1}$$

in a Hilbert space X (we shall make precise assumptions on A, F and G in Section 2).

Our point of view consists in regarding (1.1) as a perturbation of the semilinear equation

$$u' + Au = F(t,u). \tag{1.2}$$

If (1.2) is linear ($F = 0$), then there is a standard relationship between (1.1) and (1.2), given by the variation of constants formula

$$u(t) = T(t)u_0 + \int_0^t T(t-s)G(s,u(s))\,ds, \tag{1.3}$$

where $\{T(t) : t \geq 0\}$ is the linear semigroup generated by $-A$. In fact, (1.3) provides the standard definition of "mild solutions" of (1.1) (see Pazy [8]).

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