Differential and Integral Equations, Volume 5, Number 2, March 1992, pp. 387–392.

ON PAIRS OF POSITIVE SOLUTIONS FOR A CLASS OF SUB-SUPERLINEAR ELLIPTIC PROBLEMS

F.J.S.A. CORRÊA†

Departamento de Matemática e Estatística Centro de Ciências e Tecnologia
Universidade Federal da Paraíba, 58.100 - Campina Grande - PB - Brazil

(Submitted by: Peter Hess)

Abstract. We consider a class of semilinear elliptic problems where the nonlinearity f is essentially asymptotically linear. Using bifurcation theory, we prove that the problem $-\Delta u = f(x,u)$ in Ω, $u = 0$ on $\partial\Omega$, possesses at least two positive solutions if f is "sub-superlinear" among other assumptions.

1. Introduction. In this work, we consider the question of existence and multiplicity of positive solutions for the problem

$$-\Delta u = f(x,u) \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \partial\Omega, \tag{1}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded smooth domain and $f\colon \overline{\Omega} \times \mathbb{R}^+ \to \mathbb{R}$ is a locally Lipschitz continuous function which is sublinear at zero and superlinear at infinity. Throughout this paper, the following basic assumptions on f will be assumed:

- (f_1) $f(x,t) > 0$ for all $(x,t) \in \overline{\Omega} \times (0,+\infty)$;
- (f_2) $f(x,0) > 0$ or if $f(x,0) = 0$ then $0 < a_0(x) := \lim_{t\to 0^+} \frac{f(x,t)}{t}$, uniformly in $\overline{\Omega}$, and $a_0 \in C^\alpha(\overline{\Omega})$ for some $\alpha \in (0,1)$;
- (f_3) there exists a function $a_\infty \in C^\alpha(\overline{\Omega})$, for some $\alpha \in (0,1)$, such that $\lim_{t\to\infty} \frac{f(x,t)}{t} = a_\infty(x) > 0$, uniformly in $\overline{\Omega}$.

Let us denote by $\lambda_i(m)$, $i = 1,2,\ldots$, the eigenvalues of the linear eigenvalue problem

$$-\Delta u = \lambda m u \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \partial\Omega,$$

where $m \in C^\alpha(\overline{\Omega})$ and $m(x) > 0$ in Ω. It is well known that $0 < \lambda_1(m) < \lambda_2(m) \le \lambda_3(m) \le \ldots$, $\lambda_i(m) \to +\infty$, $\lambda_1(m)$ is simple and its associated eigenfunction φ_1 may be taken positive in Ω and it is the unique eigenfunction with this property.

Hereafter we assume the following:

$$\lambda_1(a_0) < 1 \quad \text{and} \quad \lambda_1(a_\infty) < 1. \tag{s}$$

Received November 1990, in revised form March 1991.
†Partially supported by CNPq/Brasil.
AMS Subject Classification: 35J25, 47H15.