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SEMILINEAR BOUNDARY VALUE PROBLEMS AT RESONANCE WITH GENERAL NONLINEARITIES

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Abstract. In this paper we prove the existence results for some semilinear boundary value problems at resonance with nonlinearities which depend on the derivatives of the solution. We consider nonlinearities for which asymptotic limits in $\pm\infty$ do not exist and also in some cases unbounded nonlinearities with sublinear growth.

1. Introduction. This paper deals with solvability of the boundary value problem (BVP)

$$-\sum_{|\alpha|\le 2m} a_\alpha(x)D^\alpha u(x) - \lambda u(x) + g(x, u(x), D^{\alpha_1}u(x), \dots, D^{\alpha_k}u(x))$$

$$= f(x), \quad x \in \Omega, \tag{1.1}$$

$$Bu(x) = 0, \quad x \in \partial\Omega, \tag{1.2}$$

where λ is the eigenvalue of symmetric differential operator

$$Au = -\sum_{|\alpha|\le 2m} a_\alpha(x)D^\alpha u(x),$$

g is nonlinear Caratheodory's function containing the partial derivatives of u of order less than or equal to $2m-1$ and B denotes the system of boundary conditions with partial derivatives of order at most $2m-1$. Let us point out that the *asymptotic limits* of g,

$$\lim_{\substack{s\to\pm\infty \\ r_i\to\pm\infty}} g(x, s, r_1, \dots, r_k),$$

need not exist and that in some particular cases (when λ is the principal and simple eigenvalue of A and the corresponding eigenfunctions do not change sign in Ω) g can be *unbounded* function with sublinear growth with respect to the variable s.

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