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NONLOCAL DISPERSIVE EQUATIONS IN WEIGHTED SOBOLEV SPACES

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Abstract. We show that the initial value problem for nonlocal dispersive equations is (locally) well-posed in Hadamard's sense in various weighted Sobolev spaces and obtain for C^∞ dispersion relations, solutions in the Schwartz space. In addition, the solutions depend continuously on the dispersion operator. As an application, we consider the intermediate long wave equation and discuss the continuity of its solution with respect to a parameter, characterizing in physical situation the depth of the fluids, and their Korteweg-de Vries and Benjamin-Ono limits. Similar results are also derived for some generalizations of the Benjamin-Bona-Mahony equation.

1. Introduction. In this work, we are concerned with the initial value problem for nonlocal dispersive equations in various weighted Sobolev spaces. Those equations take one of the following forms:

$$u_t + \mathcal{N}(u) + i\mathcal{L}(u) = 0, \tag{1.1}$$

$$u_t + \mathcal{L}(u_t) + \mathcal{N}(u) = 0, \tag{1.2}$$

where subscripts stand for partial differentiation, $i = \sqrt{-1}$, $u = u(t,x)$, $t > 0$, $x \in \mathbb{R}^n$, is a real(or complex)-valued function (in the applications, u is the real or complex amplitude or the (real) velocity of the wave), $\mathcal{N}$ is a nonlinear functional of u and its complex conjugate $\bar{u}$ (complex case) and their derivatives and $\mathcal{L}$ is a linear "pseudo-differential" operator defined by its Fourier transform:

$$\widehat{\mathcal{L}(u)}(\xi) = \omega(\xi)\hat{u}(\xi). \tag{1.3}$$

The spectrum symbol ω in (1.3) characterizes the dispersion relation between the frequency ω and the wave number ξ of the sinusoidal wave $\Psi = \exp\{i(x\cdot\xi - \omega t)\}$ which solves the linear part of the system under consideration. In dispersive waves, $\xi \to \omega(\xi)$ is a nonlinear (i.e., $\omega(\xi) \not\equiv C\cdot\xi$) real-valued function. By nonlocal dispersion we mean that ω may be of non polynomial type. Thus, (1.1) includes the following significant models (of KdV type) pertaining to the uni-directional

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