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CONTROL OF A PLATE EQUATION WITH LARGE MEMORY*

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Abstract. We prove reachability for a plate equation with large memory. Our argument is based upon a new kind of unique continuation property of a non-local equation. The main tool is a multiplier technique combined with compactness property.

0. Introduction. In this paper we shall prove reachability for a plate equation with large memory. The equation is given as

$$\begin{aligned} &u_{tt}(x,t) + \Delta^2 u(x,t) + \sum_{|\alpha|\le 2} b_\alpha(\hat{x},t)\partial_x^\alpha u(x,t) \\ &\quad + b(\hat{x},t)u_t(x,t) + \int_0^t Q(t,\sigma)\Delta^2 u(x,\sigma)d\sigma = 0 \quad \text{in } \Omega\times(0,T), \end{aligned} \tag{0.1}$$

where $x = (x_1,\cdots,x_n) \in \Omega$, Ω is a bounded open subset of R^n with smooth boundary $\partial\Omega$, $\hat{x} = (x_2,\cdots,x_n)$, $T > 0$ is given and $\partial_x^\alpha = (\frac{\partial}{\partial x_1})^{\alpha_1}\cdots(\frac{\partial}{\partial x_n})^{\alpha_n}$, $\alpha = (\alpha_1,\cdots,\alpha_n)$. Our assumptions on the coefficients are:

$$b_\alpha(\hat{x},t) \in C^1([0,T]; C^{|\alpha|}(\bar{\Omega})) \cap L^1(0,T; C^{|\alpha|+1}(\bar{\Omega})), \quad \text{for each } |\alpha| \le 2; \tag{0.2}$$

$$b(\hat{x},t) \in C^2(\bar{\Omega}\times[0,T]) \cap L^1(0,T; C^3(\bar{\Omega})); \tag{0.3}$$

$$Q(t,\sigma) \in C^3([0,T]\times[0,T]). \tag{0.4}$$

We formulate the question of reachability as follows.

Case I: Boundary Control. For any given (u_0,u_1), are there controls g_1, g_2 such that the solution of (0.1) and

$$u(x,0) = 0, \quad u_t(x,0) = 0 \tag{0.5}$$

$$u = g_1, \quad \frac{\partial u}{\partial \nu} = g_2 \quad \text{on } \partial\Omega\times(0,T) \tag{0.6}$$

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