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ON A HYPERBOLIC EQUATION OF HIGH ORDER IN A BANACH SPACE

V. V. VARLAMOV
Department of Mathematics, Moscow Engineering and Physics Institute
31, Kashirskoe Shosse, 115409, Moscow, USSR

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Abstract. The Cauchy problem for the hyperbolic equation

$$\frac{d^n}{dt^n}[u''(t) + Au(t)] + \sum_{k=0}^{n-1} \frac{d^k}{dt^k}[u''(t) + A_k u(t)] = f(t), \quad t \in [0, +\infty), \ n \geq 1,$$

with zero initial conditions is studied in a Banach space H. Here A and A_k are linear operators mapping H into H and A is the infinitesimal generator of a strongly continuous cosine family of bounded operators in H. This equation is a generalization of the equation describing the propagation of time-dependent acoustic waves in a wide class of media with dispersion and absorption. By means of the technique of strongly continuous cosine operator functions, existence and uniqueness of the solution of this problem are established.

In recent years, interest has grown in the study of wave motions in media with dispersion and absorption. The problems arising here are not only of applied but also of theoretical interest since they are, as a rule, nonclassical and rather specific. In this paper, we investigate the questions of existence and uniqueness of a solution of some hyperbolic equations of high order describing the propagation of time-dependent acoustic waves in media of the type mentioned above.

In the paper [5], P. Renno presents the equation

$$\sum_{k=0}^{n} \tau_k \frac{\partial^k}{\partial t^k}\left(\frac{\partial^2 u}{\partial t^2} - C_k^2 \Delta u\right) = 0, \tag{1}$$

where t is time, Δ is the Laplace operator in R^3, τ_k are positive constants characterizing dissipative effects (relaxation times, absorption coefficients, etc.) and $C_k > 0$ denote constant characteristic velocities depending on the properties of the material. Examples of physical phenomena which are described by Equation (1) can be found in gasdynamics [4], hydrodynamics of multiphase media [6] and linear viscoelasticity [2].

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