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GLOBAL EXISTENCE FOR A THERMODYNAMICALLY CONSISTENT MODEL OF PHASE FIELD TYPE

SONGMU ZHENG
Institute for Mathematics, Fudan University, Shanghai 200433, China

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Abstract. In a recent paper, Penrose and Fife proposed a thermodynamically consistent model of phase field type based on the idea that the value of the entropy functional can not decrease along solution paths, in agreement with what one expects from the second law of thermodynamics. It turns out that the corresponding partial differential equations become more difficult than the one studied by Caginalp and others. In this paper, we prove the global existence, uniqueness and asymptotic behavior of smooth solutions.

1. Introduction. In a recent paper, Penrose and Fife [6] proposed thermodynamically consistent models of phase field type based on the idea that the value of the entropy functional cannot decrease along solutions paths which is in agreement with what one expects from the second law of thermodynamics. It turns out that for a system with nonconserved order parameter, the order parameter φ and the absolute temperature θ satisfy the following coupled system of partial differential equations

$$\varphi_t = K_1\Big\{\kappa_1\Delta\varphi + s_0'(\varphi) + \frac{\lambda(\varphi)}{\theta}\Big\}, \tag{1.1}$$

$$\theta_t - \lambda(\varphi)\varphi_t = -M_2\Delta\Big(\frac{1}{\theta}\Big) \tag{1.2}$$

with K_1, κ_1, M_2 being positive constants and $s_0(\varphi)$ a double well function.

If we assume that the temperature remains fairly close to some "average" value θ_0 and we linearize the equations with respect to $u = \theta - \theta_0$ and ignore the dependence of $\lambda(\varphi)$ on φ, then we are led to the phase field equations

$$\tau\varphi_t = \xi^2\Delta\varphi + \varphi - \varphi^3 + 2u, \tag{1.3}$$

$$u_t + \frac{\ell}{2}\varphi_t = K\Delta u \tag{1.4}$$

studied by Caginalp [1] and others [3] (refer to Section 6 in [6]).

However, there is no reason to guarantee that the temperature is always close to a certain value. Besides, as pointed out in [6], at least in the case of the solid-liquid

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