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ALMOST-PERIODIC ATTRACTORS FOR A CLASS OF NONAUTONOMOUS REACTION-DIFFUSION EQUATIONS ON $\mathbb{R}^N$ II. CODIMENSION-ONE STABLE MANIFOLDS

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Abstract. In this paper, we initiate an investigation of the stability properties of a one-parameter family $\{\hat{u}\}$ of spatially homogeneous, time almost-periodic classical solutions to a class of nonautonomous semilinear parabolic initial value problems with Neumann boundary conditions on bounded regions Ω of $\mathbb{R}^N$. In particular, for $p \in (N, \infty)$ and for every $\hat{u} \in \{\hat{u}\}$, we construct in the Sobolev space $H^{2,p}(\Omega, \mathbb{R})$ a codimension-one local stable manifold of classical solutions of small amplitude, which thereby all stabilize exponentially rapidly around $\hat{u}$. Our method of investigation exploits the Banach algebra structure of $H^{2,p}(\Omega, \mathbb{R})$ and mainly rests upon the construction of fixed point solutions to certain nonlinear integral equations in weighted Banach spaces of exponentially decaying $H^{2,p}(\Omega, \mathbb{R})$-valued maps. The class of equations which we analyze here contains, in particular, Fisher's type reaction-diffusion equations of population genetics. The results of this paper are thereby complementary to those of [14] and [15].

1. Introduction and outline. This is the second of a series of articles devoted to the analysis of stabilization phenomena for certain classical solutions to real semilinear parabolic Neumann boundary value problems of the form

$$\begin{cases} u_t(x,t) = \Delta u(x,t) + s(t) g(u(x,t)), \quad (x,t) \in \Omega \times \mathbb{R}^+, \\ \operatorname{Ran}(u) \subseteq (u_0, u_1), \\ \dfrac{\partial u}{\partial \underset{\sim}{n}}(x,t) = 0, \quad (x,t) \in \partial\Omega \times \mathbb{R}^+ \end{cases} \tag{1.1}$$

([14]–[16]). In equations (1.1), Ω denotes an open bounded connected subset of $\mathbb{R}^N$ with smooth boundary $\partial\Omega$ and $N \in [2, \infty) \cap \mathbb{N}^+$, while Δ stands for Laplace's operator in the x-variables. Furthermore, $s\colon \mathbb{R}^+ \to \mathbb{R}$ is the restriction to $\mathbb{R}^+$ of a Bohr almost-periodic function on $\mathbb{R}$ which we shall also denote by s, while $g \in \mathcal{C}^{(1)}(\mathbb{R}, \mathbb{R})$ possesses at least two zeroes u_0 and u_1 such that $g(u) > 0$ for every $u \in (u_0, u_1)$ with the property that $g'(u_0) > 0$ and $g'(u_1) < 0$. Finally, $\operatorname{Ran}(u)$ denotes the range of u and $\underset{\sim}{n}$ stands for the normalized outer normal vector to $\partial\Omega$.

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