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THE MONOTONICITY METHOD AND FREE VIBRATIONS OF EXTENSIBLE BEAMS

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Abstract. We prove the existence of infinitely many nontrivial time-periodic solutions (free vibrations) for the equation

$$Cu_{tt} - u_{ttxx} + u_{xxxx} + f_1(x,u,u_x) - f_2(x,u,u_x)_x - \int_0^\pi u_z^2(z,t)\,dz u_{xx} = 0$$

complemented with the boundary conditions $u(0,t) = u(\pi,t) = u_{xx}(0,t) = u_{xx}(\pi,t) = 0$. The problem may be rewritten in the form $\tilde{\mathcal{L}}u + \tilde{f}(u) = 0$, where $\tilde{\mathcal{L}}^{-1}/R(\tilde{\mathcal{L}})$ is a compact linear operator and $\tilde{f}$ is pseudomonotne in the sense of Browder. Then the variational approach is used to prove the results.

This paper is intended to be a contribution to the vast amount of results concerning the nontrivial (nonzero) solutions to the functional equations $\mathcal{L}u + f(u) = 0$, where $\mathcal{L}$ is a linear and f a nonlinear operator on an appropriate abstract space. The general form of the equation allows us to consider various problems of mathematical physics. Thus, for instance, the existence of time-periodic solutions to nonlinear systems of second order ordinary differential equations or of Hamiltonian systems (see [2], [11]), the bound states of nonlinear elliptic equations ([1], [4]), or even some semilinear hyperbolic problems ([5], [9], [10]) may be tackled successfully by means of the methods sketched below.

We confine ourselves to the case where the underlying problem is represented by a "hyperbolic" partial differential equation complemented by a suitable set of boundary conditions.

A truly pioneering work in this direction is a paper [10] of Rabinowtiz concerning the time-periodic solutions to semilinear wave equations. Rabinowitz, as well as many of his followers, assumes that $\mathcal{L}$ possesses a compact generalised inverse on the range space $R(\mathcal{L})$ while the lack of compactness due to the infinite dimensional nullspace $\ker(\mathcal{L})$ is compensated for by the monotonicity of the substitution operator f.

In the present paper, we are interested in the problem

$$Cu_{tt} - u_{ttxx} + u_{xxxx} + f_1(x,u,u_x) - f_2(x,u,u_x)_x - \int_0^\pi u_z^2(z,t)\,dz u_{xx} = 0, \qquad \text{(E)}$$

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