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ON EXTREMAL SOLUTIONS OF AN ELLIPTIC BOUNDARY VALUE PROBLEM INVOLVING DISCONTINUOUS NONLINEARITIES

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1. Introduction. Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with a boundary $\partial\Omega$ of class C^1. In this paper we shall consider the following boundary value problem (BVP)

$$-Lu = F(u,u) \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega, \tag{1.1}$$

where F is the superposition operator associated with a function $f\colon \Omega\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ by $F(u,v)(x) = f(x,u(x),v(x))$, and L is assumed to be an uniformly elliptic linear operator of the form

$$Lu = \frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial u}{\partial x_j}\right) - b_i\frac{\partial u}{\partial x_i},$$

with coefficients a_{ij}, $b_i \in L^\infty(\Omega)$, $i,j = 1,\dots n$.

The aim of this paper is to show the existence of the greatest and the smallest solutions of the BVP (1.1) lying between upper and lower solutions. In the case when the function f above is independent of its third argument and is of Carathéodory type this question has been considered by Dancer and Sweers in [4], and for more general equations by Kura in [10] (see also the references therein). The existence of solutions between upper and lower solutions can be proved in this case under very weak regularity conditions on the data (cf. [5], [10]). However, the proof of the existence of extremal solutions, as done in [4], [10], requires stronger regularity assumptions. To be more precise, in [4] the coefficients of the linear operator L must be smooth enough to ensure solutions to lie in the Sobolev space $W^{2,1}_{loc}(\Omega)$, such that one can apply an inequality due to Kato (cf. [9]). In Kura's paper (see [10], Theorem 3.2) the local boundedness as well in the growth conditions as for the W–upper and W–lower solutions is needed.

In the present paper we do not need any of the above mentioned regularity restrictions. Moreover, the BVP (1.1) may also involve discontinuous nonlinearities, since we assume that the function $(x,r)\to f(x,r,s)$ is of Carathéodory type for each

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