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REMARKS ON THE NUMBER OF POSITIVE SOLUTIONS FOR A CLASS OF NONLINEAR ELLIPTIC PROBLEMS*

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1. Introduction. The effect of the topology of the domain on the existence of more positive solutions for elliptic problems has been approached by several authors, see [4], [7], [8], who deal, above all, with the Sobolev critical exponent. In particular, in a recent work [5], V. Benci and G. Cerami have studied the problem

$$\begin{cases} -\Delta u + \lambda u = |u|^{p-1}u & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is an open smooth bounded subset of $\mathbb{R}^N$, $N \geq 3$, $\lambda \in \mathbb{R}^+$ and $1 < p < \frac{N+2}{N-2}$, and have proved that there exists a link between the number of solutions of (1.1) and the topology of Ω.

The aim of this note is to extend the results of [5] to a class of problems such as

$$\begin{cases} -\Delta u + \lambda u = f(u) & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.2}$$

where $f : \mathbb{R} \to \mathbb{R}$ and Ω and λ are as in (1.1). More exactly we prove that if f satisfies the following conditions:

(f1) f is C^2;

(f2) $\exists a > 0$, $b > 0$ such that $at^p \leq f(t) \leq bt^p$, $\forall t \geq 0$, where $1 < p < \frac{N+2}{N-2}$ and

$$1 < \frac{b}{a} < 1 + \varepsilon^*, \quad \text{with } \varepsilon^* = \sup\left\{\varepsilon > 0 : 1 + \frac{2}{p+1}(1+\varepsilon)^{\frac{2}{p-1}} > (1+\varepsilon)^{\frac{p+1}{p-1}}\right\};$$

(f3) $\exists\ a', b', a'', b''$ positive constants such that

$$|f'(t)| \leq a' + b'|t|^{p-1}, \quad \forall t \geq 0, \quad |tf''(t)| \leq a'' + b''|t|^{p-1}, \quad \forall t \geq 0;$$

(f4) $\frac{d}{dt}\left(\frac{1}{t}f(t)\right) > 0, \quad \forall t > 0, \quad f''(0) = 0;$

(f5) put $F(t) = \int_0^t f(s)\,ds$ for every $t \in \mathbb{R}$; there exists $\vartheta > 0$ such that

$$\vartheta \leq \frac{1}{2} - \frac{1}{2}\left(\frac{b}{a}\right)^{\frac{2}{p-1}}\left(\frac{b}{a} - \frac{2}{p+1}\right) \quad \text{and} \quad F(t) \leq \vartheta t f(t);$$

then the following theorem holds:

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