Differential and Integral Equations, Volume 5, Number 3, May 1992, pp. 521–535.

OPTIMAL REGULARITY, EXACT CONTROLLABILITY AND UNIFORM STABILIZATION OF SCHRÖDINGER EQUATIONS WITH DIRICHLET CONTROL*

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Abstract. We first identify the space of optimal regularity of a Schrödinger equation defined on a smooth bounded domain $\Omega \subset \mathbb{R}^n$ with $L_2(0,T;L_2(\Gamma))$ – nonhomogeneous term (control) in the Dirichlet boundary conditions. Next, we prove exact controllability and uniform stabilization results on this optimal space, the latter via an explicit dissipative feedback operator. As a consequence of these results, the abstract theory of the optimal quadratic cost problem over an infinite horizon and related Algebraic Riccati Equation as in [3] is applicable to this Schrödinger mixed problem. This, in particular, provides another stabilizing feedback operator, generally non-dissipative, defined in terms of the corresponding Riccati operator.

1. Introduction, preliminaries, and statement of main results. Let Ω be an open, bounded domain in $\mathbb{R}^n$, where typically $n \geq 2$, with sufficiently smooth boundary $\Gamma = \Gamma_0 \cup \Gamma_1$, Γ_i relatively open with Γ_0 possibly empty, and Γ_1 nonempty. In Ω we consider the following mixed problem for the Schrödinger equation in $y(t,x)$:

$$y_t = -i\Delta y \qquad \text{in } (0,T] \times \Omega = Q \tag{1.1a}$$

$$y(0,\,\cdot\,) = y_0 \qquad \text{in } \Omega \tag{1.1b}$$

$$y\big|_{\Sigma_0} \equiv 0 \qquad \text{in } (0,T] \times \Gamma_0 = \Sigma_0 \tag{1.1c}$$

$$y\big|_{\Sigma_1} = u \qquad \text{in } (0,T] \times \Gamma_1 = \Sigma_1 \tag{1.1d}$$

with control function u in the Dirichlet B.C. In this paper we first identify (§2) the space of optimal regularity of problem (1.1) with $u \in L_2(0,T;L_2(\Gamma_1)) = L_2(\Sigma_1)$. Next, we give results of exact controllability (§3) and uniform stabilization (§4) for (1.1) on such space. As a consequence of these results, the abstract theory of the optimal quadratic cost problem over an infinite horizon and related Algebraic Riccati Equation as in [3] is applicable to this Schrödinger mixed problem; see Remark 1.4 below. Besides being of interest in itself, the Schrödinger equation may

Received for publication April 1991.
*Research partially supported by National Science Foundation under Grant DMS-87-96320 and by the Air Force Office of Scientific Research under Grant AFOSR-87-0321.
AMS Subject Classifications: 35, 93.