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CONVERGENCE THEOREMS FOR INTEGRATED SEMIGROUPS

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Abstract. Theorems giving the continuous dependence of an integrated semigroup on its infinitesimal generator are proved. These results generalize the Trotter and Trotter-Kato theorems for C_0 semigroups. They are useful in the study of the convergence of numerical approximations of certain hyperbolic initial value problems which can be formulated as abstract Cauchy problems and which naturally lead to integrated semigroups. The relation of these results with the corresponding classical semigroup theorems are discussed.

1. Introduction. Cauchy problems in abstract spaces often generate semigroups when the initial data satisfy sufficient regularity conditions. Such problems are commonly written in the form

$$\frac{du}{dt} = Au, \quad u(0) = u_0,$$

where A is a closed operator on the space X. When u_0 is not sufficiently regular, there may not exist a strong solution $u(t) \in X$ but, following a germinal idea of Da Prato and Sinestrari [3], the integrated equation

$$u(t) = u_0 + A\int_0^t u(s)\,ds$$

can admit what they term an integral solution $\int_0^t u(s)\,ds$, which can belong to the domain of A even when $u(s)$ does not. An abstract framework for such integral solutions has been developed by Arendt [1], Kellerman and Hieber [4], Neubrander [5] and Thieme [7]. In the terminology of Arendt [1], the integrated Cauchy problem generates an integrated semigroup in a precise setting that we will describe in the next section. Thieme [7] gives a detailed discussion of the wave operator in $\mathbb{R}^m$ with initial data in $L^2(\mathbb{R}^m)$ and shows that it generates an integrated semigroup, while the data has to satisfy additional regularity restriction in order for it to generate a semigroup.

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