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ON THE EQUIVALENCE OF GREEN FUNCTIONS OF SECOND ORDER ELLIPTIC EQUATIONS IN $\mathbb{R}^n$

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Abstract. Let P be a second order elliptic operator which is defined on $\mathbb{R}^n$ and assume that P admits a positive minimal Green function $G_P(x,y)$ in $\mathbb{R}^n$. We prove that $G_P(x,y)$ and $G_{-\Delta}(x,y)$ are equivalent provided that the coefficients of P satisfy certain decay conditions at infinity. By equivalent we mean that there exists a positive constant C such that $C^{-1} \leq G_P(x,y)/G_{-\Delta}(x,y) \leq C$ for all $x, y \in \mathbb{R}^n$, $x \neq y$. We indicate that our decay assumptions are optimal. The proof relies on pointwise estimates of the iterated Green kernel. As an application, we prove a Liouville type theorem.

1. Introduction. Let P be a second order uniformly elliptic operator on $\mathbb{R}^n$, $n \geq 3$, with sufficiently smooth coefficients. Assume that P is either of the form

$$Pu = -\sum_{i,j=1}^{n} a_{ij}(x)\partial_i\partial_j u + \sum_{i=1}^{n} b_i(x)\partial_i u + c(x)u \tag{1.1}$$

or P is in a divergence form

$$Pu = -\sum_{i,j=1}^{n} \partial_i(a_{ij}(x)\partial_j u) + \sum_{i=1}^{n} b_i(x)\partial_i u + c(x)u, \tag{1.2}$$

where $x = (x_1, \dots, x_n) \in \mathbb{R}^n$ and $\partial_i = \partial/\partial x_i$. Assume also that P is *subcritical* in $\mathbb{R}^n$; that is, P admits a positive minimal Green function $G_P(x,y)$ in $\mathbb{R}^n$. Let

$$P_\infty u = -\sum_{i,j=1}^{n} a_{ij}(\infty)\partial_i\partial_j u, \tag{1.3}$$

where $\big(a_{ij}(\infty)\big)$ is a given positive definite matrix. Denote also

$$P_0 u = -\sum_{i,j=1}^{n} \partial_i(a_{ij}(x)\partial_j u). \tag{1.4}$$

The main purpose of this paper is to show that there exists a positive constant C that depends only on the coefficients of P such that

$$C^{-1}|x-y|^{2-n} \leq G_P(x,y) \leq C|x-y|^{2-n}, \tag{1.5}$$

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