Differential and Integral Equations, Volume 5, Number 4, July 1992, pp. 831–854.

SMOOTH SYMMETRY BREAKING BIFURCATION FOR FUNCTIONAL DIFFERENTIAL EQUATIONS*

P. DORMAYER
Mathematisches Institut der Universität München
Theresienstraße 39, 8000 München 2, West - Germany

(Submitted by: Roger Nussbaum)

Abstract. We consider the differential delay equation $\dot{x}(t) = -\alpha f(x(t-1))$, where f is an odd C^2-map with $f'(0) < 0$. This equation has a primary branch $PB(f)$ of periodic solutions which have the symmetry $x(t+2) = -x(t)$, $t \in \mathbb{R}$. We examine the characteristic multipliers of these solutions and give conditions for the existence of bifurcation points on $PB(f)$ from which a smooth curve of nonsymmetric periodic solutions bifurcates. Finally, we show that given $n \in \mathbb{N}$, there is a $f \in C^2(\mathbb{R})$ which has at least n smooth bifurcation points.

1. Introduction. Let $f \in C^2(\mathbb{R})$ be an odd map and consider the differential delay equation

$$\dot{x}(t) = -\alpha f(x(t-1)). \qquad (\alpha f)$$

It is well known that this equation has a smooth primary branch of special symmetric solutions. These solutions are uniquely parameterized by the amplitude and fulfill the special symmetry

$$x(t+2) = -x(t), \quad t \in \mathbb{R}.$$

In [2,4], we proved that for $f(x) = \frac{x}{1+x^2}$, $x \in \mathbb{R}$, the primary branch contains a bifurcation point from which a smooth curve of periodic solutions of (αf) bifurcates. These solutions have the symmetry

$$x(t+\tau) = -x(t), \quad t \in \mathbb{R},$$

for some $\tau \neq 2$.

Thus, the question arises if symmetry breaking secondary bifurcation is possible for this class of functional differential equations. This was shown by Walther [7]. However, two problems remain.

- Walther did not prove whether the bifurcating solutions lye on a smooth curve; he only found periodic solutions arbitrary near to the bifurcation point.
- Numerical studies for $f = \sin$ suggest that there is a sequence $\alpha_k > 0$, $k \in \mathbb{N}_0$, of symmetry breaking bifurcation points with $\alpha_k \to \infty$ and $\alpha_{k+1} - \alpha_k \to \pi$ for $k \to \infty$.

In this paper, we will answer the first question. More precisely we have:

Received for publication May 1991.
*This paper was supported by the Deutsche Forschungsgemeinschaft.
AMS Subject Classification: 34K15.