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CONVERGENCE RESULTS FOR HAMILTON-JACOBI-BELLMAN EQUATIONS IN VARIABLE DOMAINS

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Abstract. We study the stability of viscosity solutions of Dirichlet problems for Hamilton-Jacobi equations with respect to perturbations of the domain where the equation is set, of the boundary data and the Hamiltonian. We give estimates showing that the rate of convergence of the solutions depends only on the rate of convergence of the data and on the exponent of Hölder continuity of the solutions at the boundary of the domain. We apply the result to the eikonal equation of geometric optics and to the Bellman-Isaacs equations of some optimal control problems and differential games, including the classical pursuit-evasion and minimum-time problems.

0. Introduction. We are interested in Dirichlet problems for Hamilton-Jacobi equations of the form

$$\begin{cases} u + H(x, u, Du) = 0 & \text{in } \mathcal{O}, \\ u = g & \text{on } \partial\mathcal{O}, \end{cases} \tag{P}$$

where $\mathcal{O}$ is an arbitrary open set and H and g are at least continuous. We consider perturbations of the set $\mathcal{O}$, the Hamiltonian H and the boundary data g, whose size is measured by a parameter $h > 0$, and our goal is estimating how far is the solution u_h of the perturbed problem from the solution u of (P), in terms of the parameter h. By solution we will always mean viscosity solution, following Crandall and P.L. Lions [10], Crandall, Evans and Lions [8] and Lions [15]. It is well known that these solutions enjoy very strong stability properties. Our main point here is the continuity of solutions with respect to the domain, in the sense of the convergence of $\mathbb{R}^N \setminus \mathcal{O}$ in the Hausdorff topology, and an estimate of the rate of convergence.

Our motivation comes from optimal control problems and differential games. It is known that the value functions of such problems satisfy in the viscosity sense

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