Differential and Integral Equations, Volume 5, Number 4, July 1992, pp. 777–792.

NONLINEAR SCALAR FIELD EQUATIONS

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Abstract. We prove existence results for a class of semilinear elliptic differential equations in $\mathbb{R}^N$ ($N \geq 3$). The nonlinearities contain sub- and supercritical exponents, and the assumptions for the coefficients are rather general. Moreover, we state some conditions so that the solutions decay exponentially.

1. Introduction and presentation of the results. In the present paper, we consider the nonlinear eigenvalue problem

$$-\Delta u - q(x)|u|^{\sigma_1}u + r(x)|u|^{\sigma_2}u = \lambda u \quad \text{in } \mathbb{R}^N, \tag{1.1}$$

where $N \geq 3$, $0 < \sigma_1 < 4/(N-2)$ and $\sigma_2 \geq 4/(N-2)$.

The nontrivial solutions of equation (1.1) supply standing waves for nonlinear Klein-Gordon and Schrödinger equations. In the case that q and r are positive constants, this equation has been studied by W.A. Strauss [11] (see Example 2) and by H. Berestycki and P.-L. Lions [3] (see also Example 2). These authors were motivated by a paper of D. Anderson [1] who considered the case $N = 3$, $\sigma_1 = 2$ and $\sigma_2 = 4$.

In the following, we require that the functions q and r satisfy the conditions (A)–(D) or (A_r)–(D_r).

(A) The functions q, $r\colon \mathbb{R}^N \to \mathbb{R}$ are measurable and r satisfies $r(x) \geq r_0$ almost everywhere in $\mathbb{R}^N$, where r_0 is a positive constant.

(B) There exist an open ball $B \subset \mathbb{R}^N$ with $B \neq \emptyset$ and $0 \notin \overline{B}$ and a sequence of real numbers (t_k) satisfying

$$1 = t_1 < t_2 < \cdots < t_k < t_{k+1} < \ldots$$

and $t_k \to \infty$ ($k \to \infty$), so that

$$q(x) \geq f(x)|x|^{\sigma_1((N/2)-1)-2} \quad \text{holds for almost all } x \in \mathcal{B},$$

where $\mathcal{B} = \bigcup_{k=1}^{\infty} B_k$, $B_k = t_k B$ and $f\colon \mathcal{B} \to [0,\infty)$ is a measurable function satisfying

$$\gamma_k = \operatorname*{ess\,inf}_{x \in B_k} f(x) \to \infty \quad (k \to \infty).$$

Received for publication in revised form August 1991.
AMS Subject Classification: 35P30, 35J20, 35D05.