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A PROBABILISTIC APPROACH TO VOLTERRA EQUATIONS IN BANACH SPACES

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Abstract. This paper presents a probabilistic approach to the Volterra equations considered in [7]. Our aim is to establish the transformation formula from a solution of one equation to that of another equation. The probabilistic approach should serve to deepen our understanding of the transformation formula. As a special case of this transformation formula, we derive the probabilistic expression of the solution.

1. Introduction. Let $W(dt)$ be the measure on $[0,\infty)$ so that $W([0,t])$ satisfies the integral equation

$$aW([0,t]) + \int_0^t k(t-s)W([0,s])\,ds = t, \quad t \geq 0, \tag{1.1}$$

where $a \geq 0$ and $k \in L^1_{\mathrm{loc}}([0,\infty))$ is nonnegative and nonincreasing with $k(0+) = \infty$ if $a = 0$. We are concerned with the following Volterra equation in a real Banach space X:

$$u(t) = \phi + \int_{[0,t]} Au(t-s)W(ds), \quad t \geq 0, \tag{1.2}$$

where A is the infinitesimal generator of a linear C_0-semigroup $\{T_t\}_{t\geq 0}$ on X and $\phi \in D(A)$. The Volterra equation (1.2) is a special case studied in [7] (see (1.2) and (3.1) of [7]). Although more general equations were studied in [7], we treat only the equation (1.2) in this paper.

For $j = 1, 2$, let $W_j(dt)$ be the measure of (1.1) for some a_j and $k_j(t)$. Our aim is to establish the transformation formula from a solution of (1.2) for $W = W_1$ to a solution of (1.2) for $W = W_2$ (Theorem 1). Here we fix A and ϕ. To achieve this aim, we adopt the probabilistic approach, which should serve to deepen our understanding of the transformation formula. The clue to this approach is that we can express explicitly the measure $W(dt)$ of (1.1) by a subordinator (Proposition). Then we explain the transformation formula by the subordination of subordinators. As a special case of Theorem 1, we derive the probabilistic expression of the solution of (1.2) (Theorem 2). This expression is a generalization of the result obtained in [11].

The present paper is organized as follows: we shall state the main results in §2 and prove them in §3.

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