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RESONANCE OSCILLATIONS OF WAVE EQUATIONS

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Abstract. Resonance oscillations of wave equations are studied and sufficient conditions are given that the amplitude of the solution of the boundary value problem grows up. The approach used is to reduce the multi-dimensional problem to a one-dimensional problem.

In this paper we consider the wave equation

$$\frac{\partial^2 u}{\partial t^2} + \sigma \frac{\partial u}{\partial t} - \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\big(a_{ij}(x)\frac{\partial u}{\partial x_j}\big) + c(x)u = f(x,t), \quad (x,t) \in \Omega, \tag{1}$$

and the boundary condition

$$u = \psi \quad \text{on } \partial G \times [0,\infty), \tag{2}$$

where G is a bounded regular domain in $\mathbb{R}^n$ and $\Omega = G \times (0,\infty)$.

Forced oscillations of the wave equation (1) were studied by several authors; see, for example, [2, 3, 5]. Our objective is to study the resonance oscillation of the wave equation (1). Here, the resonance oscillation means that the amplitude of the solution of the boundary value problem (1), (2) grows up if the forcing term $f(x,t)$ and the boundary value ψ are periodic functions of t with the same period and the frequency approaches some frequency which is characterized by (1) (cf. Timoshenko, Young and Weaver [4]). It seems that little is known about the resonance of the equation (1).

We assume throughout this paper that

(A_1) σ is a nonnegative constant;
(A_2) $f(x,t) \in C(\bar{\Omega})$ and $\psi \in C(\partial G \times [0,\infty))$;
(A_3) there exists an open domain $G_0 \supset \bar{G}$ with the property that
 (i) $a_{ij}(x) \in C^{1,\alpha}(G_0)$, $\alpha > 0$, $a_{ij}(x) = a_{ji}(x)$ and there exists a positive constant μ such that

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq \mu \sum_{i=1}^{n} \xi_i^2 \quad \text{for all } \xi \in \mathbb{R}^n,\ x \in G_0;$$

 (ii) $c(x) \in C^{0,\alpha}(G_0)$ and $c(x) \geq 0$ in G_0.

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