Differential and Integral Equations, Volume 5, Number 6, November 1992, pp. 1383–1405.

SEMILINEAR EVOLUTION EQUATIONS WITH SINGULARITIES IN ORDERED BANACH SPACES

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(Submitted by: M.G. Crandall)

Abstract. A class of semilinear evolution equations of second order in time are treated in ordered Banach spaces. The evolution equations include nonlinear operators which represent singular nonlinearities and the class is an abstraction of a class of systems of ordinary and partial differential equations with singular nonlinear terms. A notion of mild solution to the initial-value problem for a semilinear equation of the class is introduced and the regularity of the mild solutions is investigated. Since the evolution equations are formulated in ordered Banach spaces, a comparison theorem for the mild solutions can be obtained and two existence theorems are established through the method of successive approximations. Asymptotic behaviors of the mild solutions are also discussed with the aid of the comparison theorem and some new results on systems of ordinary differential equations with polynomial singularities. These results are applied to weakly coupled systems of singular hyperbolic equations.

This paper is concerned with the construction and asymptotic properties of solutions of a typical class of semilinear evolution systems with singular nonlinearities. In order to treat systems of ordinary or partial differential equations with singular nonlinear terms it is often effective to interpret them as evolution equations in ordered function spaces. A prototype of such equations is the semilinear hyperbolic equation of a single space variable with a polynomial singularity of the form

$$u_{tt} + \phi(t,x)u^{-\lambda} = \kappa^2 u_{xx}, \qquad t > 0,\ -\infty < x < \infty, \tag{1}$$

where $\lambda > 0$, $\kappa \geq 0$ and ϕ is a nonnegative continuous function on $[0,\infty) \times \mathbb{R}$ satisfying appropriate smoothness and growth conditions. We here study a class of systems of semilinear evolution equations of the form

$$u''(t) + B_\alpha(t)u(t) = Au(t), \quad t > 0 \tag{2}$$

in an ordered Banach space $(X, |\cdot|, \geqq)$. Here $u''(t) = Au(t)$ is an abstraction in X of linear wave equations and $B_\alpha(t)$ is a nonlinear operator from a closed set $X_\alpha \equiv \{v \in X : v \geqq \alpha e\}$ into the positive cone $X_0 \equiv \{v \in X : v \geqq 0\}$, where e is supposed to be a nonnegative element of X such that $w \geqq \alpha e$ for some $\alpha > 0$

Received for publication Feburary 6, 1991.

Partially supported by a Grant-in-Aid for Scientific Research from the Japan Ministry of Education.

AMS Subject Classifications: 35L70, 47H07, 47H15.