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EXISTENCE RESULTS FOR SOME NONLINEAR PARABOLIC EQUATIONS WITH NONREGULAR DATA

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Abstract. We prove existence and regularity theorems for some nonlinear parabolic equations of the form

$$u_t + A(u) = f$$

in a bounded cylinder Q, where A is an operator of the Leray-Lions type. Here the datum f is a bounded Radon measure or an L^m function (with m "small") so that the "standard" variational setting does not apply.

1. Introduction and statement of results. In this paper, we will consider the following parabolic equation:

$$\begin{cases} u_t - \operatorname{div} a(x,t,u,\nabla u) = f \text{ in } Q \\ u(x,0) = u_0(x) \text{ for a.e. } x \in \Omega \\ u(x,t) = 0 \text{ for } (x,t) \in \Gamma. \end{cases} \tag{P}$$

Here Ω is a bounded open set in $\mathbb{R}^N$, $N \geq 2$, Q is the cylinder $\Omega \times (0,T)$, where T is a real positive number, and Γ is the "lateral surface" $\partial\Omega \times (0,T)$.

The operator $A(u) = -\operatorname{div} a(x,t,u,\nabla u)$ is an operator of the Leray-Lions type (see [9]). We will study the existence of a solution for (P) under various hypotheses on the data f and u_0. The difficulty lies in the fact that we will not choose these data in a "classical" dual space (for instance, f will be a bounded measure), so that it will not be possible to use the variational framework (see [8]).

To solve this problem, the following two steps, which are, in a way, "classical," are needed:

— *a priori* L^q-regularity results for the gradients of solutions of (P);

— approximation of f with regular functions and study of the convergence of the solutions of the corresponding problems, using the estimates to prove that the limit is a solution of (P).

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