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ON SOME CONDITIONS FOR EXISTENCE OF FORCED PERIODIC OSCILLATIONS*

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Abstract. This paper is concerned with some sufficient conditions for the existence of forced periodic oscillations for systems of ordinary differential equations. The method introduced here generalizes some ideas of the method of guiding functions for the study of periodic problems under conditions which allow some blow-up of the solutions.

1. Introduction. Consider the system of ordinary differential equations

$$x' = f(t,x), \quad x \in \mathbb{R}^n, \tag{1}$$

with the right-hand side T-periodic in t,

$$f(t+T,x) = f(t,x), \quad t \in \mathbb{R},\ x \in \mathbb{R}^n,$$

and satisfying the Caratheodory conditions. No conditions will be made about the uniqueness of solutions of the Cauchy problem for (1) or about the non-local continuability of such solutions. The Euclidian norm and inner product in $\mathbb{R}^n$ will be denoted by $|\cdot|$ and $(\cdot,\cdot)$, respectively. We shall assume the existence of a real function V on $\mathbb{R}^n$ satisfying the following conditions.

Assumption (V). There exists a positive function V of class C^1 on $\mathbb{R}^n$, a number $r_0 > 0$ and a nonnegative Caratheodory function $a : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ such that

$$\lim_{|x|\to\infty} V(x) = \infty \tag{2}$$

and

$$(V'(x), f(t,x)) \le a(t, V(x)), \quad |x| \ge r_0. \tag{3}$$

Such functions are analogues of the guiding functions we considered in [3] in the case where $a(t,v) = a(t)$. The reader can consult [1] and [3] for more references on

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