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INTERMEDIATE SCHAUDER THEORY FOR SECOND ORDER PARABOLIC EQUATIONS IV: TIME IRREGULARITY AND REGULARITY

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According to the classical Schauder theory of parabolic equations [4, 9], when the coefficients and inhomogeneous term in a second order parabolic equation are Hölder continuous with respect to space and time, so are the second space and first time derivatives. When this assumed Hölder continuity is only with respect to space (uniformly in time), almost the same regularity is true as before. Using maximum principle arguments, Brandt [1] showed that the second space derivatives are Hölder in space and then Knerr [8] proved their Hölder continuity in time. From the equation we infer that the first time derivative is also Hölder in space. (In general this derivative will be discontinuous in time as the equation $u_t = u_{xx} + f(t)$ with discontinuous f shows.) These results are valid only away from the parabolic boundary of the domain in which the equation is assumed to hold. Brandt mentions a forthcoming paper on boundary regularity which does not seem to have appeared in print.

More recently Sinestrari and Von Wahl [24] used semigroup theory to study such equations when the coefficients are time-independent and the inhomogeneous term is Hölder in space and continuous in time. Unlike Brandt and Knerr, Sinestrari and von Wahl prove various global regularity results assuming sufficient smoothness of the boundary data in a cylindrical domain.

Our goal here is to reproduce these and related results in a unified fashion. The hypotheses will be intermediate between those of Brandt and Knerr and those of Sinestiari and von Wahl: the coefficients and inhomogeneous term will be Hölder in space and bounded and measurable in time. In fact, for our boundary estimates we will sometimes allow unboundedness of some of these quantities near the boundary using spaces like those in the series [11, 12, 15], of which this paper forms a part. We shall follow the notation, definition and section numbering of that series. Section 15 gives a proof of the interior regularity results of Brandt and Knerr via Campanato space theory, which yields space and time regularity simultaneously. Boundary and consequent global estimates for the first initial-boundary value problem in general appropriate smooth (cylindrical or non-cylindrical) domains appear in Section 16. Then the initial-oblique derivative problem is studied in Section 17; our results are more general than those of Pogorzelski [20-23] as described in [4, Chapter 5].

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