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NONLINEAR SECOND-ORDER ELLIPTIC EQUATIONS WITH JUMP DISCONTINUOUS COEFFICIENTS PART I: QUASILINEAR EQUATIONS

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Abstract. This is the first paper of a series of two devoted to the study of quasilinear or fully nonlinear, second-order, uniformly elliptic equations with coefficients which are smooth on both sides of a surface without necessarily being equal on that surface. We prove that C^1 solutions exist and are unique with appropriate conditions on the nonlinearities and regularity on each side of the surface. Here, we only consider the case of quasilinear equations. The case of fully nonlinear elliptic equations will be considered in Part II.

1. Introduction. In this paper and its sequel (Part II), we investigate the Dirichlet problem for quasilinear or fully nonlinear, second-order, uniformly elliptic equations

$$a^{k,ij}(x,u,Du)u_{x_ix_j} + a^k(x,u,Du) = 0 \quad \text{in } \Omega_k, \quad k=1,2, \quad u=g \text{ on } \partial\Omega \tag{1}$$

or

$$f^k(x,u,Du,D^2u) = 0 \quad \text{in } \Omega_k, \quad k=1,2, \quad u=g \text{ on } \partial\Omega \tag{2}$$

in a bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$, with a smooth boundary $\partial\Omega$. We assume that $\Omega = \Omega_1 \cup \Omega_2 \cup S$ is divided into two subdomains Ω_1 and Ω_2 by a smooth surface S without self-crossing points. The coefficients of (1), (2) are supposed to be smooth on each side of S on $\overline{\Omega}_1$, $\overline{\Omega}_2$, respectively, and, when considered as function on Ω, present some pure jump discontinuity on S.

We wish to show here and in Part II that, under general and natural conditions on the coefficients including uniform ellipticity, there exists a unique solution of (1) or (2) which belongs at least to $C^1(\Omega) \cap C(\overline{\Omega})$. We adopt this rather vague formulation in order to avoid distinguishing between (1) and (2). In the case of (1), for general nonlinearities f^1, f^2, the equation will be understood in a viscosity sense (see the survey paper [8]); viscosity solutions, as introduced by M.G. Crandall and P.L. Lions [5] and M.G. Crandall, L.C. Evans and P.L. Lions [6], have provided a general and efficient tool for studying the existence, uniqueness and regularity questions for fully nonlinear elliptic equations and some references include [16-18],

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