Differential and Integral Equations, Volume 5, Number 5, September 1992, pp. 1121–1150.

UNIFORM DECAY RATES FOR THE SOLUTIONS TO THE EULER-BERNOULLI PLATE EQUATION WITH BOUNDARY FEEDBACK ACTING VIA BENDING MOMENTS

MARY ANN HORN†

Department of Applied Mathematics, University of Virginia, Charlottesville, Virginia 22903

(Submitted by: A.V. Balakrishnan)

Abstract. The aim of this paper is to consider a physically valid and meaningful model of the Euler-Bernoulli plate with boundary conditions which include moments of inertia realistically present in the system. For this model, we shall prove that the appropriately selected *boundary* feedback uniformly stabilizes the model, i.e., the energy decays to zero uniformly when $t \to \infty$. Our choice of boundary feedback is motivated by the fact that boundary controls are easily implemented as the need to act only on the bundary of the spatial domain. Inclusion of the moments of inertia in the model is a source of serious mathematical difficulties when deriving the appropriate estimates for stabilization. We shall need, as a preliminary step of analysis, to develop new regularity results for the traces of the solution to special types of plate problems. These regularity results together with sharp energy estimates allow us to prove that uniform stabilization holds.

1. Introduction.

1.1. Statement of the problem. Let Ω be an open bounded domain in $\mathbb{R}^2$ with a sufficiently smooth boundary $\Gamma = \Gamma_0 \cup \Gamma_1$, with Γ_0 possibly empty and Γ_1 non-empty. In Ω, we consider the following model of the Euler-Bernoulli plate with homogeneous Dirichlet boundary conditions and a control, u, acting through the bending moments:

$$w_{tt} + \Delta^2 w = 0 \qquad \text{in } Q_T = (0,T) \times \Omega \tag{1.1.a}$$

$$\left.\begin{aligned} w(0,\cdot) &= w_0 \\ w_t(0,\cdot) &= w_1 \end{aligned}\right\} \qquad \text{in } \Omega \tag{1.1.b}$$

$$w = 0 \qquad \text{on } \Sigma_T = (0,T) \times \Gamma \tag{1.1.c}$$

$$\begin{cases} \Delta w + (1-\mu)Bw = 0 & \text{on } \Sigma_T^0 = (0,T) \times \Gamma_0 \\ \Delta w + (1-\mu)Bw = u \in L_2(\Sigma_T^1) & \text{on } \Sigma_T^1 = (0,T) \times \Gamma_1, \end{cases} \tag{1.1.d}$$

where the boundary operator, B, in (1.1d) takes the form:

$$Bw = -\frac{\partial^2 w}{\partial \tau^2} - k\frac{\partial w}{\partial \nu} = -k\frac{\partial w}{\partial \nu}. \tag{1.2}$$

Received March 1991.

†Supported under a National Science Foundation Graduate Fellowship.

AMS Subject Classification: 35, 93.