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CONVEX CONTROL PROBLEMS FOR HEREDITARY SYSTEMS

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Abstract. This work is concerned with the dynamic programming method (variational method) for convex control problems with the hereditary state system. After obtaining the Hamilton-Jacobi equation which is satisfied by the optimal value function, the dual Hamilton-Jacobi equation is calculated. In the last part of the paper, we study the special quadratic case for which the corresponding Riccati equations are obtained.

Let us consider the Hamilton-Jacobi equation

$$\begin{cases} \psi_t(t,y_1,y_2) + F(B^*\psi_{y_1}(t,y_1,y_2)) - (A_0 y_1 + A_1 y_2(-a), \psi_{y_1}(t,y_1,y_2)) \\ \qquad\qquad -\left(\frac{dy_2}{ds}, \psi_{y_2}(t,y_1,y_2)\right) = g(y_1) \\ \psi(0,y_1,y_2) = \psi_0(y_1), \end{cases} \tag{1}$$

where $(t,y_1,y_2) \in [0,T] \times \mathbb{R}^n \times W^{1,2}(-a,0;\mathbb{R}^n)$ and the following hypotheses are satisfied:

(i) $F\colon \mathbb{R}^m \to \mathbb{R}$ is a continuous convex function which is bounded on bounded subsets;

(ii) $A_0\colon \mathbb{R}^n \to \mathbb{R}^n$ is a linear operator and A_1 and B are matrices;

(iii) $\psi_0,\ g\colon \mathbb{R}^n \to \mathbb{R}$ are continuous convex functions which are bounded on bounded subsets.

We have denoted by ψ_t, ψ_{y_1}, ψ_{y_2} the partial derivatives (in a generalized sense) of the function ψ with respect to t, y_1 and y_2, respectively. Denote by $\mathcal{K}$ the set of all convex functions $\psi \in C(\mathbb{R}^n \times W^{1,2}(-a,0;\mathbb{R}^n))$. We associate the function $h\colon \mathbb{R}^m \to \mathbb{R}$, defined by

$$h(u) = \sup\{-(p,u) - F(p) : p \in \mathbb{R}^m\},$$

i.e., $h(u) = F^*(-u)$, to the function F. Then the assumption (i) is equivalent to the condition $\lim_{|u|_U\to\infty} \frac{h(u)}{|u|_U} = \infty$.

Definition 1. The function $\psi\colon [0,T] \times \mathbb{R}^n \times L^2(-a,0;\mathbb{R}^n) \to \mathbb{R}$ defined by

$$\begin{aligned} \psi(t,y_1,y_2) = \inf\Big\{ &\int_0^t (g(x(s)) + h(u(s)))\,ds + \psi_0(x(t)) : \\ &x'(t) = A_0 x(t) + A_1 x(t-a) + Bu(t) \text{ in } [0,T],\ x(0) = y_1,\ x(s) = y_2(s) \\ &\text{for a.e. } s, -a \le s \le 0,\ u \in L^2(0,T;\mathbb{R}^m)\Big\} \end{aligned}$$

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