Differential and Integral Equations, Volume 5, Number 5, September 1992, pp. 1017–1032.

QUALITATIVE BEHAVIOUR OF AN EPIDEMICS MODEL

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(Submitted by: Peter Hess)

Abstract. We investigate convergence and stability properties of a system of two differential equations which stems from a model of man environment diseases such as cholera or typhus. The first equation is a parabolic equation. Under the assumption that the human population is sedentary, the second equation will be an ordinary differential equation so that the compactness of the solution operator is lost. The two equations are coupled via a boundary feedback operator. First we show that this system fits in the theory of abstract parabolic equations. To get sub- and supersolutions for the system, we then investigate an eigenvalue problem associated to a family of elliptic boundary value problems. Finally, results on discrete strongly order preserving dynamical systems are generalized to a class of noncompact mappings in order to apply them to the equation with T-periodic and sublinear nonlinearities.

1. Introduction. In this paper, we investigate existence and asymptotic behaviour of solutions of a system of reaction-diffusion equations which consist of a parabolic equation and an ordinary differential equation which are coupled via a boundary feedback operator and some nonlinearity. The system stems from a model for man environment diseases such as cholera or typhus and was introduced in [4] (see also the references given there). We write the system in the following form:

$$\begin{cases} \partial_t u_1 + \mathcal{A} u_1 = g_1(x,t,u) & \text{in } \Omega \times (0,\infty), \\ \partial_t u_2 = g_2(x,t,u) & \text{in } \overline{\Omega} \times (0,\infty), \\ \mathcal{B} u_1 = K u_2 & \text{on } \partial\Omega \times (0,\infty), \\ u(0) = v & \text{in } \Omega \quad , \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with boundary $\partial\Omega$ of class C^2, $u = (u_1, u_2)$, (g_1, g_2) is a given nonlinearity and v is an initial condition. Here

$$\mathcal{A} = - \sum_{1 \le i,j \le n} a_{ij} \partial_{ij} + \sum_{1 \le i \le n} a_i \partial_i + a_0$$

is a strongly elliptic operator with coefficients lying in $C(\overline{\Omega})$ and

$$\mathcal{B} = \partial_\beta + b_0$$

Received for publication August 1991.

AMS Subject Classifications: 35B, 92D30, 35K65.