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UNIQUENESS FOR VISCOSITY SOLUTIONS OF QUASILINEAR ELLIPTIC EQUATIONS IN $\mathbb{R}^N$ WITHOUT CONDITIONS AT INFINITY

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Abstract. By means of suitable interior estimates, uniqueness for continuous viscosity solutions of equations like

$$-\Delta u + H(\nabla u) + \lambda u^m = f \quad \text{in } \mathbb{R}^N$$

are obtained, provided $f \in C(\mathbb{R}^N)$, $\lambda > 0$, $m > 1$ and H is uniformly continuous. Then the classical Perron method is used in proving the existence whenever $H(0) \leq f(\cdot)$ in $\mathbb{R}^N$.

I. Introduction. In the framework of the very weak solutions, H. Brezis [2] showed the uniqueness of the problem

$$-\Delta u + u^m = f \quad \text{in } \mathcal{D}'(\mathbb{R}^N), \quad u \in L^1_{\text{loc}}(\mathbb{R}^N), \quad m > 1.$$

In some sense, a similar goal is sought for the quasilinear

$$-\Delta u + H(x, u, \nabla u) = 0 \quad \text{in } \mathbb{R}^N \tag{$\mathcal{E}$}$$

by means of the viscosity solutions. This kind of solution is a continuous function satisfying ($\mathcal{E}$) in an adequate sense which uses the second order semijets (see Section II for precise definitions). Viscosity solutions were introduced in several papers of M.G. Crandall and P.L. Lions which focused on the relative first order notion. For the original reference, see [5]. (See also the paper written jointly with L.C. Evans [3]). Recently, Crandall, Ishii and Lions have collected part of their related works on viscosity solutions in the monograph [4].

We note that several models arising in the applications only admit continuous solutions. For instance, it occurs in some PDE's which are the dynamic programming of certain optimal control problems. Then the viscosity solution theory applies.

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