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SOME BASIC FACTS FOR A CLASS OF DEGENERATE PARABOLIC EQUATIONS

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Abstract. In this paper, we establish some results concerning comparison, existence of initial traces and uniqueness of solutions of equations of the form $u_t = \Delta\phi(u) + f$, where ϕ is a continuous strictly increasing function.

1. Introduction. In this paper, we are interested in "parabolic" equations of the form

$$u_t = \Delta\phi(u) + f, \tag{1.1}$$

where $\phi\colon \mathbb{R} \longrightarrow \mathbb{R}$ is nondecreasing. We will establish some results concerning local smooth approximation, comparison and existence in the context of initial-boundary value problems and existence of initial traces for the Cauchy problem of a sort which were used by Huan [9] in extending results of Dahlberg and Kenig [5]. It is expected that these results, which are of independent interest, will be helpful in further extensions of the theory.

We are, in particular, interested in the problem of "initial traces". This problem is the following: given a solution u of (1.1) on some strip $\{(x,t) : x \in \mathbb{R}^N,\ 0 < t < T\}$, can we assert that $\lim_{t\downarrow 0} u(\cdot,t) = u_0$ exists uniquely in some space and topology? If so, is u uniquely determined by u_0? The results known in this direction involve restrictions on u (e.g., $u \geq 0$), very severe restrictions on ϕ and typically assume $f \equiv 0$; they typically assert that u_0 is well-defined in the space of measures and uniquely determines u. Here we deal with rather general ϕ and f at the cost of assuming relatively strong conditions on u. One of our main results is:

Theorem 1.1. *Let $T > 0$, $f \in L^1(\mathbb{R}^N \times (0,T))$ and $\phi : [0,\infty) \to [0,\infty)$, $\phi(0) = 0$, be bounded on bounded sets and satisfy*

$$\limsup_{r\downarrow 0} \frac{\phi(r)}{r^\theta} < \infty, \tag{1.2}$$

for some $0 < \theta \leq 1$. Let

$$u \in L^\infty((0,T), L^1(\mathbb{R}^N)) \cap L^\infty(\mathbb{R}^N \times (\epsilon,T)), \quad \epsilon > 0,$$

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