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ASYMPTOTIC BEHAVIOUR OF POSITIVE SOLUTIONS OF THE NONAUTONOMOUS LOTKA-VOLTERRA COMPETITION EQUATIONS*

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1. Introduction. In this paper, we study the asymptotic behaviour of positive solutions of the system

$$\begin{aligned} u' &= u[a(t) - b(t)u - c(t)v], \\ v' &= v[d(t) - e(t)u - f(t)v], \end{aligned} \tag{0.1}$$

where $a, \dots, f \in C_+$ and C_+ is the set of all continuous functions $g\colon \mathbb{R} \to \mathbb{R}$ bounded above and below by positive constants.

When $a, \dots, f$ are constant, system (0.1) is the well known Lotka-Volterra model of competition between two species. The periodic (resp. almost periodic) case has been considered in [3] and [6] (resp. [2] and [5]). The general case has been considered in [1], "since few things in nature are truly periodic."

Actually, we study the asymptotic behaviour of positive solutions of a class of Lotka-Volterra systems which are "asymptotic" to (0.1). See Theorem 0.1 below.

To be precise, let C^o be the space of all bounded and continuous functions $g\colon \mathbb{R} \to \mathbb{R}$. For g in C^o, we define

$$g_L = \inf\{g(t)\colon t \in \mathbb{R}\}, \quad g_M = \sup\{g(t)\colon t \in \mathbb{R}\},$$
$$g_L(\infty) = \liminf_{t\to\infty} g(t) \ \text{ and } \ g_M(\infty) = \limsup_{t\to\infty} g(t).$$

From now on, we write $B = b/a$, $C = c/a$, $E = e/d$ and $F = f/d$, and we prove the following results.

Theorem 0.1. *Let (u_*, v_*) be a positive solution of the system*

$$\begin{aligned} u' &= u[a_*(t) - b_*(t)u - c_*(t)v], \\ v' &= v[d_*(t) - e_*(t)u - f_*(t)v], \end{aligned} \tag{0.1$_*$}$$

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