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SOME REMARKS ON THE EXACT CONTROLLABILITY OF HYPERBOLIC SYSTEMS WITH MIXED BOUNDARY CONDITIONS

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Abstract. In this paper, we adapt the H.U.M. (Hilbert Uniqueness Method) of J.L. Lions [6] to the case of mixed boundary conditions (i.e., Dirichlet-Neumann boundary conditions) when the control is a Dirichlet action. In the numerical part, we apply the method developed in A. Eljendy [2] to a transmission line problem and we illustrate the efficiency of the method by numerical results.

1. Introduction. The aim of this paper is to study the exact controllability problem for the mixed boundary control (i.e., Dirichlet-Neumann condition) in the case where the control is a Dirichlet action. Let Ω be a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\Gamma = \partial\Omega$ or with a convex polygonal (or polyhedral) boundary $\Gamma = \partial\Omega$. Let us consider the system

$$\begin{cases} u'' = a\Delta u + bu' + cu & \text{in } Q =]0,T[\times\Omega \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \Sigma^N =]0,T[\times\Gamma^N \\ u = v & \text{on } \Sigma^D =]0,T[\times\Gamma^D \\ u(0) = u^0, \;\; u'(0) = u^1 & \text{in } \Omega, \end{cases} \tag{1.1}$$

where a, b and c are constants, $a > 0$, $b \leq 0$ and $c \leq 0$, $u' = \frac{\partial u}{\partial t}$, $u'' = \frac{\partial^2 u}{\partial t^2}$, $\frac{\partial u}{\partial \nu}$ denotes the normal derivative, $\Gamma = \Gamma^N \cup \Gamma^D$, and v is the control function. By the change of variable $u = e^{\frac{b}{2}t}y$, the system (1.1) is equivalent to the system

$$\begin{cases} y'' - \Delta y + \alpha y = 0 & \text{in } Q \\ \frac{\partial y}{\partial \nu} = 0 & \text{on } \Sigma^N \\ y = v & \text{on } \Sigma^D \\ y(0) = y^0, \;\; y'(0) = y^1 & \text{in } \Omega, \end{cases} \tag{1.2}$$

where α is real number. The paper is organized as follows. In Section 2, we consider the case $\alpha = 0$. Sufficient conditions for the exact controllability are given. In Section 3, we construct some examples which satisfy the sufficient conditions. In Section 4, we treat the case $\alpha \neq 0$. The exact controllability is also proved in the

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