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INTEGRAL CONTINUITY AND STABILITY FOR STOCHASTIC HYPERBOLIC EQUATIONS

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Abstract. Continuous dependence of mild solutions to stochastic hyperbolic problems on coefficients is investigated. Fairly general integral continuity theorems are obtained provided the state space is endowed with an appropriately weakened norm. In the course of the proofs, some results on asymptotic stability are established which are of independent interest.

1. Introduction and preliminaries. This paper aims at investigating the continuous dependence of solutions to a stochastic hyperbolic equation

$$\frac{\partial^2 u}{\partial t^2}(t,x) + Lu(t,x) = f(t,u(t,x)) + g(t,u(t,x))\xi(t,x) \tag{1}$$

on coefficients f, g. Here $(t,x) \in [0,T] \times B$, $B \subseteq \mathbb{R}^d$ is an open domain, $-L$ is, say, a second order elliptic differential operator, ξ is some "state-dependent noise," and we add appropriate boundary and initial conditions to (1). We wish to obtain results of integral continuity type in which the topology on the coefficients is given by the convergence of their indefinite integrals. (A precise description will be given later.) We see two reasons for investigating these types of theorems. First, they seem to be fairly general. As established by Z. Artstein, the integral continuity theorems represent in a certain precisely definable sense the best possible result on the continuous dependence of solutions to an ordinary differential equation

$$\dot{x} = f(t,x)$$

on the right hand side (see [1], cf. also the paper [9]). Second, integral continuity theorems stemmed from the effort to find an abstract framework for averaging procedures for finding asymptotical solutions to equations with rapidly oscillating coefficients (cf. [8], [14]). We illustrate such an application at the end of Section 2 of this paper.

Equation (1) is purely formal. Wanting to employ the semigroup approach to stochastic partial differential equations, we treat (1) as an equation

$$\begin{aligned} d\begin{pmatrix} u \\ u_t \end{pmatrix} &= \left[\begin{pmatrix} 0 & I \\ -L & 0 \end{pmatrix}\begin{pmatrix} u \\ u_t \end{pmatrix} + \begin{pmatrix} 0 \\ f(t,u) \end{pmatrix}\right] dt + \begin{pmatrix} 0 \\ g(t,u) \end{pmatrix} dw(t) \\ &\equiv \left[\mathfrak{L}\begin{pmatrix} u \\ u_t \end{pmatrix} + \begin{pmatrix} 0 \\ f(t,u) \end{pmatrix}\right] dt + \begin{pmatrix} 0 \\ g(t,u) \end{pmatrix} dw(t) \end{aligned} \tag{2}$$

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