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ON REGULARLY VARYING SOLUTIONS OF SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS

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Abstract. Asymptotic representations are given for regularly varying solutions of the equation $y'' = f(x)y$ for positive f.

1. Introduction and results. We consider positive decreasing solutions of the equation

$$y'' = f(x)y \tag{1.1}$$

assuming that f is positive and continuous on a half axis $[a, \infty)$. In that case, such a solution always exists. The second linearly independent one, and hence all solutions, can be treated by the usual Wronskian technique.

In order to state our results, we need the following function class. A measurable function $L\colon \mathbb{R}^+ \to \mathbb{R}$ which is eventually positive is said to be slowly varying at infinity ($L \in RV_0$) if

$$\lim_{x\to\infty} \frac{L(tx)}{L(x)} = 1 \quad \text{for all} \quad t > 0.$$

Also, the function

$$\psi(x) = x^\alpha L(x) \tag{1.2}$$

is said to be regularly varying of index α ($\psi \in RV$ or $\psi \in RV_\alpha$).

This class has been extensively used in various branches of analysis and probability theory and for its basic properties the reader is referred to [1], [2]. It is shown in [4] that all positive decreasing solutions of (1.1) are in the class $RV_{-\alpha}$ ($\alpha \geq 0$) if and only if there exists $c \geq 0$ such that for $x \to \infty$,

$$\phi(x)\colon = x \int_x^\infty f(t)\,dt - c \to 0, \tag{1.3}$$

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