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ON THE VIBRATIONS OF RECTANGULAR MEMBRANES*

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Abstract. We consider the wave equation in a rectangular domain Ω with homogeneous Dirichlet boundary condition. Let S be an open segment in Ω, parallel to one of the sides of Ω. We show that if S is shorter than the corresponding side, then for every (arbitrary large) positive number T, there exist smooth initial data such that the corresponding solution of the system is strictly positive in every point of S during the whole time interval $(-T, T)$.

1. Introduction. Let Ω be a rectangular domain in $\mathbb{R}^2$, say

$$\Omega = \left(0, \frac{\pi}{A}\right) \times \left(0, \frac{\pi}{B}\right),$$

let us denote its boundary by Γ, and consider in Ω the wave equation with homogeneous boundary conditions:

$$\begin{aligned} &u'' - \Delta u = 0 && \text{in } \Omega \times \mathbb{R}, \\ &u = 0 && \text{on } \Gamma \times \mathbb{R}, \\ &u(0) = u^0 \text{ and } u'(0) = u^1 && \text{in } \Omega. \end{aligned} \tag{1.1}$$

It is well known that for every $(u^0, u^1) \in H_0^1(\Omega) \times L^2(\Omega)$, the system (1.1) has a unique solution satisfying

$$(u, u') \in C(\mathbb{R}; H_0^1(\Omega) \times L^2(\Omega)).$$

The solution cannot keep a constant sign unless $u^0 = u^1 = 0$. More precisely, if $(u^0, u^1) \neq (0, 0)$ and if I is an interval of length $|I| > \pi/\sqrt{A^2 + B^2}$, then the solution of (1.1) takes both (strictly) positive and negative values on subsets of positive measure of $\Omega \times I$. This is a simple special case of a theorem of Cazenave and Haraux [3], valid for arbitrary bounded domains Ω in $\mathbb{R}^n$ and for semilinear wave equations.

On the other hand, it was observed in [5] that the pointwise behavior of the solutions is quite different. For some particular choices of A, B and of a point

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