

ON THE GEOMETRY OF DEGREE CONDITIONS FOR MULTIPARAMETER OSCILLATION THEOREMS†

P. BINDING

University of Calgary, Department of Mathematics and Statistics
Calgary, Alberta T2N 1N4, Canada

H. VOLKMER

Universität Gesamthochschule Essen, Fachbereich Mathematik
Universitätsstr. 3, 4300 Essen 1, Germany

(Submitted by: F.V. Atkinson)

Abstract. We give various sufficient conditions for the existence of eigenvalues of Sturm-Liouville systems involving k spectral parameters. We begin with a degree condition which leads us to a new sufficient condition weaker than right definiteness. We also interpret our conditions geometrically in the special case of three parameters.

1. Introduction. The theory of multiparameter oscillation theorems goes back to an investigation of Klein [12] in 1881. Subsequent authors Bôcher [8], Ince [10], Binding and Browne [3], Volkmer [18] et al. generalized his result which today is known as Klein's oscillation theorem. This theorem concerns a k -parameter Sturm-Liouville system of the form

$$\begin{aligned} (p_r y_r')' + q_r y_r + \sum_{s=1}^k \lambda_s a_{rs}(x_r) y_r &= 0, \quad 0 \leq x_r \leq 1, \quad r = 1, \dots, k, \\ \alpha_{r1} y_r(0) + \alpha_{r2} y_r'(0) &= 0, \quad \beta_{r1} y_r(1) + \beta_{r2} y_r'(1) = 0, \quad r = 1, \dots, k, \end{aligned} \quad (1.1)$$

where we shall assume that the given functions $p_r, q_r, a_{rs}, r, s = 1, \dots, k$, are real-valued and continuous on the interval $[0, 1]$. Moreover, p_r is continuously differentiable and positive on $[0, 1]$ for each r . The vectors $(\alpha_{r1}, \alpha_{r2})$ and (β_{r1}, β_{r2}) are real and nonzero for each r .

An *eigenvalue* of (1.1) is a column $\lambda = [\lambda_1 \dots \lambda_k]^T$ of real numbers such that the differential equations and the boundary conditions of (1.1) can be satisfied simultaneously by eigenfunctions y_1, \dots, y_k none of which vanishes identically. If y_r has n_r zeros in $]0, 1[$, then $\mathbf{n} = (n_1, \dots, n_k)$ is the oscillation count corresponding to λ . By an *oscillation theorem* for (1.1) we mean a result which gives existence of

Received for publication November 1991.

†The research was supported by NSERC of Canada.

AMS Subject Classification: 49G05.