Differential and Integral Equations, Volume 6, Number 3, May 1993, pp. 705–719.

BOUNDARY VALUE PROBLEMS OF A CLASS OF QUASILINEAR ORDINARY DIFFERENTIAL EQUATIONS

ZONGMING GUO

Department of Mathematics, University of Glasgow, G12 8QW UK

Department of Mathematics, Henan Normal University, Xinxiang, 453002, P. R. China

(Submitted by: Jean Mawhin)

Abstract. We consider a class of quasilinear ordinary differential equations. Using the generalized degree theory, we establish the existence of C^1-solutions for periodic and Neumann boundary value problems.

1. Introduction. In this paper, we establish the existence of solutions to the periodic boundary value problem (BVP)

$$(|u'|^{p-2}u')' + f(t,u,u') = y(t), \quad u(0) = u(1),\ u'(0) = u'(1) \tag{1.1}$$

under various conditions on the function $y : [0,1] \to \mathbb{R}$ and the function $f : [0,1] \times \mathbb{R}^2 \to \mathbb{R}$.

We also consider the problem of the following form

$$\left.\begin{aligned} &Au - f(t,u) = 0 \quad \text{in } (0,1) \\ &u'(0) = u'(1) = 0, \end{aligned}\right\} \tag{1.2}$$

where $Au = -(a(|u'|^2)u')'$, $a : \mathbb{R} \to \mathbb{R}$ is a continuous mapping such that $h(t^2) = \int_0^{t^2} a(\tau)\,d\tau$ is a strictly convex function on $\mathbb{R}$. That is, the equation (1.2) coincides with the equation $g'(u) = 0$, where

$$g(t) = \frac{1}{2}\int_0^1 h(|u'|^2)\,dt - \int_0^1 \int_0^{u(t)} f(\tau, u(\tau))\,d\tau\,dt. \tag{1.3}$$

By a C^1-solution of problem (1.1) we mean that $u \in C^1([0,1])$, $u(0) = u(1)$, $u'(0) = u'(1)$ and u satisfies

$$|u'(t)|^{p-2}u'(t) - |u'(0)|^{p-2}u'(0) = -\int_0^t [f(s,u(s),\,u'(s)) - y(s)]\,ds.$$

By a C^1-solution of problem (1.2) we mean that $u \in C^1([0,1])$ satisfying $u'(0) = u'(1) = 0$ and

$$-(a|u'|^2)u')' - f(t,u) = 0 \quad \text{a.e. in } [0,1].$$

Received May 1991, in revised form November 1991.

AMS Subject Classification: 34B15, 47H15.